Internet Appendix for
"Social Security and Trends in Wealth Inequality"

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ABSTRACT

In this appendix, we give a detailed account of the methodology described in Section III. We explain the construction of our dataset to allow for replication and explain our discount rate assumptions. We then describe the adjustments we make to reflect life expectancy differences, early/late retirement choices, and benefit adjustments for those who receive survivor benefits, or do not receive benefits at all. Next, we explain the steps to constructing heterogeneous discounts rates used in Section C. Finally, we provide a lengthy discussion of the steps followed to assign simulated Social Security wealth to individuals.

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A. Survey of Consumer Finances

We use the triennial Survey of Consumer Finances for two main purposes: (i) measuring marketable wealth shares, and (ii) estimating aggregate Social Security wealth, and determining the share of Social Security wealth going to the wealthy. The SCF is well suited to measure marketable wealth shares as it covers most asset classes and provides detailed information on households' liabilities. We measure marketable wealth using the net worth variable: the sum of assets net of liabilities.

A. Raw SCF

Social Security benefits To study Social Security in the SCF, we collect several variables from the raw SCF data which are listed below. We report the variable name for the second person in the household (typically the spouse) in parentheses.

- `X5306` (`X5311`): Social Security benefit amount. Note that these are reported at different frequencies.
- `X5307` (`X5312`): Social Security benefit frequency. The variable values and their corresponding frequencies are as follows: 4) monthly, 5) quarterly, 6) annually, 12) every two months, -7) other, 0) no benefits.
- `X5304` (`X5309`): Social Security benefit type. This variable takes three values, which represent three benefit categories: 1) retirement, 2) disability, and 3) survivor.
- `X5305` (`X5310`): Number of years receiving Social Security benefits.
- `X19`: Age of second person.
- `X103`: Gender of second person.

From these we create a series of variables. First, we create a payment frequency variable, given by

$$\texttt{pay\_freq} = \begin{cases} 12 & \text{if } \texttt{X5307} \ (\texttt{X5312}) = 4 \\ 4 & \text{if } \texttt{X5307} \ (\texttt{X5312}) = 5 \\ 1 & \text{if } \texttt{X5307} \ (\texttt{X5312}) = 6 \\ 2 & \text{if } \texttt{X5307} \ (\texttt{X5312}) = 11 \\ 6 & \text{if } \texttt{X5307} \ (\texttt{X5312}) = 12 \\ 0 & \text{otherwise} \end{cases}$$

which allows us to calculate annual benefits, given by

$$\texttt{ssinc} = \begin{cases} \texttt{X5306} * \texttt{pay\_freq} & \text{if Head of Household} \\ \texttt{X5311} * \texttt{pay\_freq} & \text{if Second Person in Household.} \end{cases}$$

We further subdivide this income by benefit type, with retirement income given by

$$\texttt{ssinc\_ret} = \begin{cases} \texttt{ssinc} & \text{if } \texttt{X5304} \ (\texttt{X5309}) = 1 \\ \texttt{ssinc} & \text{if } \texttt{X5304} \ (\texttt{X5309}) = 2 \ \& \ \texttt{age} \ (\texttt{X19}) \geq 62 \end{cases}$$

and observed survivor benefits given by

$$\texttt{ssinc\_ben} = \texttt{ssinc} \quad \text{if } \texttt{X5304} \ (\texttt{X5309}) = 3.$$

Note that the second condition for retirement benefits assigns disability benefits going to people of retirement age as retirement benefits, consistent with the SSA. Finally, we calculate the age at retirement, which is given by

$$\texttt{ret\_age} = \begin{cases} \texttt{age} - \texttt{X5305} & \text{if Head of Household} \\ \texttt{X19} - \texttt{X5310} & \text{if Second Person in Household} \end{cases}$$

and is used to calculate full retirement age benefits in Section B.

Wage income To perform the individual assignment of Social Security wealth, we also gather data on individual income from the SCF. In particular, we gather:

– X4112 (X4712): Pre-tax wage income from primary source. Note that these are reported at different frequencies.

– X4113 (X4713): Pre-tax wage income from primary source frequency. The variable values and their corresponding frequencies are as follows: 1) daily, 2) weekly, 3) bi-weekly, 4) monthly, 5) quarterly, 6) annually, 11) semi-annually, 12) every two months, 18) hourly, 31) twice per month, -7) other, 0) no benefits.

– X4110 (X4710): Hours worked in normal week for primary source.

– X4509 (X5109): Pre-tax wage income from secondary source. Note that these are reported at different frequencies.

– X4510 (X5110): Pre-tax wage income from secondary source frequency. The variable values and their corresponding frequencies are as follows: 1) daily, 2) weekly, 3) bi-weekly, 4) monthly, 5) quarterly, 6) annually, 11) semi-annually, 12) every two months, 18) hourly, 31) twice per month, -7) other, 0) no benefits.

– X4507 (X5107): Hours worked in normal week for secondary source.

We frequency adjust the reported wage income to obtain annual individual income for individuals, given by

$$\texttt{wage\_pay\_freq} = \begin{cases} 365 & \text{if } \texttt{X4112} \ (\texttt{X5312}) = 1 \\ 52 & \text{if } \texttt{X4112} \ (\texttt{X5312}) = 2 \\ 26 & \text{if } \texttt{X4112} \ (\texttt{X5312}) = 3 \\ 12 & \text{if } \texttt{X4112} \ (\texttt{X5312}) = 4 \\ 4 & \text{if } \texttt{X4112} \ (\texttt{X5312}) = 5 \\ 1 & \text{if } \texttt{X4112} \ (\texttt{X5312}) = 6 \\ 2 & \text{if } \texttt{X4112} \ (\texttt{X5312}) = 11 \\ 6 & \text{if } \texttt{X4112} \ (\texttt{X5312}) = 12 \\ \text{Hours} \times 52 & \text{if } \texttt{X4112} \ (\texttt{X5312}) = 18 \\ 24 & \text{if } \texttt{X4112} \ (\texttt{X5312}) = 31 \\ 0 & \text{otherwise} \end{cases} .$$

From this we take annual wages as

$$\texttt{wage\_annual} = \texttt{X4112} \times \texttt{wage\_pay\_freq}.$$

These are done for each income source.

Household debt We also collect data on household interest rates and loan amounts for different types of debt which are used to construct the heterogeneous discount rates from Section C. The loan types and accompanying variables are:

- **Mortgages:** Balance outstanding — X805, X905, X1005; Annual interest rate — X816, X916, X1016.

- **Other property loans:** Balance outstanding — `X1044`; Annual interest rate — `X1045`.
- **Lines of credit:** Balance outstanding — `X1108`, `X1119`, `X1130`; Annual interest rate — `X1111`, `X1122`, `X1133`.
- **Home improvement loans:** Balance outstanding — `X1215`; Annual interest rate — `X1216`.
- **Real estate investment and vacation property loans:** Balance outstanding — `X1715`, `X1815`, `X1915`; Annual interest rate — `X1726`, `X1826`, `X1926`.
- **Auto loans:** Balance outstanding — `X2218`, `X2318`, `X2418`, `X7169`; Annual interest rate — `X2219`, `X2319`, `X2419`, `X7170`.
- **Non-auto vehicle loans:** Balance outstanding — `X2519`, `X2619`; Annual interest rate — `X2520`, `X2620`.
- **Other consumer loans:** Balance outstanding — `X2723`, `X2740`, `X2823`, `X2840`, `X2923`, `X2940`; Annual interest rate — `X2724`, `X2741`, `X2824`, `X2841`, `X2924`, `X2941`.

These data are used to obtain the value-weighted interest rate on debt for all households.

B. Cleaned SCF Extract

All wealth variables come from the cleaned SCF extract data. In particular, the `networth` variable is used to calculate the wealth distribution in each survey. This variable includes all assets less debt given in the SCF. We add to this the wealth held by the Forbes 400 as listed in the replication code of Saez and Zucman (2016). The SCF does not survey people beyond a certain wealth threshold, so people in the Forbes 400 are excluded from the sample. To fill this gap, we add aggregate Forbes 400 to the aggregate wealth of the Top 0.01%. In addition, we also add the aggregate value of defined benefit pensions into the SCF using data from the

Distributional Financial Accounts (DFA) provided by the Federal Reserve Board of Governors (Batty et al., 2019). The DFA provide data on the dollar value of the stock of defined benefit pension obligations going to the top 0.1%, the rest of the top 1%, the 90th-99th percentile, the 50th-90th percentile, and the bottom 50%. To incorporate these data to our results, we add the appropriate number to our aggregate wealth of the top 1%, top 10%, and bottom 90% in all results focusing on wealth inequality. Finally, results in the main text are reported at the individual level. This means that when reporting wealth shares, we split two person households and assign equal `networth` to each member. The results are nearly identical when creating wealth shares at the household level.

We also calculate a liquid wealth variable which is used to construct Figure 9. The component pieces of this variable are as follows:

- `liq`: liquid accounts, which is the sum of all checking, savings, and money market accounts, call accounts at brokerages, and prepaid cards.
- `cds`: certificates of deposit.
- `nmmf`: directly held mutual funds.
- `stocks`: wealth held in stocks.
- `bond`: wealth held in bonds of any type excluding savings bonds.
- `savbnd`: savings bonds.

From these, liquid wealth is given by

$$\texttt{liquid\_wealth} = \texttt{liq} + \texttt{cds} + \texttt{nmmf} + \texttt{stocks} + \texttt{bond} + \texttt{savbnd}. \tag{IA1}$$

We also add two additional components to construct a measure of quasi-liquid wealth. These pieces are as follows:

– `retqliq`: quasi-liquid retirement accounts, which are the sum of IRAs, thrift-type accounts, current pensions, and future pensions.

– `homeeq`: home equity, which is the value of the home less the outstanding mortgage principal.

From these, quasi-liquid wealth is given by

$$\texttt{quasi\_liquid\_wealth} = \texttt{liquid\_wealth} + \texttt{retqliq} + \texttt{homeeq}. \tag{IA2}$$

Finally, it is important to note that the Raw SCF values are in nominal terms (e.g. the 1995 Raw SCF is in 1995 dollars) while the Cleaned SCF Extract are in the dollars of the most recent survey year (e.g. 2019 dollars at the time of this writing). The SCF adjusts the Cleaned SCF Extract using the Consumer Price Index for all urban consumers (CPI-U-RS) from the Bureau of Labor Statistics. To make the two datasets consistent, we adjust the Cleaned SCF Extract to nominal dollars.

B. Assumptions and adjustments

A. *Market implied vs. SSA yield curve assumptions*

Appendix Figure IA5 shows the differences in the yield curve assumptions implied from Treasury notes and the assumptions used by the SSA to compute the present value of Social Security obligations. The SSA discount rates are based on historical business cycles rather than market-implied rates, which is erroneous given the persistence of the current low interest rate environment.[†] An additional piece of evidence of the issues with the SSA's approach comes from the Federal Reserve, which reported in December, 2019 FOMC meeting projections that

[†]Summers, Lawrence, "U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound," *Business Economics*, 2014, *49* (2).

median long-run nominal rates are expected to be around 2.4-2.8%, with an upper bound of 3.3%, significantly below the 5+% suggested by the SSA.

B. Full retirement benefits

To validate the simulation methodology, we compare benefits in the simulated and SCF data. In reality, individuals can choose to retire early or delay retirement, meaning we must adjust their benefits in the data to compare them with benefits implied by the simulation. Beneficiaries retiring before the full retirement age receive reduced benefits, while beneficiaries retiring after the full retirement age receive increased benefits. Therefore, we define individual i's *full retirement benefit* as

$$\text{Full Retirement Benefit}_i = \frac{\text{Benefit}_i}{\text{Adjustment}}$$

where the adjustment term depends on the number of years that the beneficiary retires early or late.

For beneficiaries retiring early, the discount is 5/9% for each month before the full retirement age, up to 36 months, and 5/12% for each additional month. For beneficiaries retiring late, the amount of the credit depends of the beneficiary's birth year and can be found here. Further, the full retirement age is different for each cohort and can be found here. From these data, we create the `full_retirement_age` variable allowing us to determine the number of years of early or late retirement as

$$\texttt{ret\_discount\_years} = \texttt{full\_retirement\_age} - \texttt{ret\_age}.$$

This variable allows us to compute the appropriate benefit adjustment.

Here is an example to help clarify the procedure: Take a 62 year retiring in 2019. This person was born in 1957, meaning that the full retirement age for her cohort is 66 years and 6 months old. For this person, we have Adjustment $= (1 - \frac{5}{9} \cdot 36 - \frac{5}{12} \cdot 18)$, meaning that the full retirement benefit is given by

$$\text{Full Retirement Benefit}_i = \frac{\text{Benefit}_i}{(1 - \frac{5}{9} \cdot 36 - \frac{5}{12} \cdot 18)}.$$

In this case, the observed benefit is adjusted upward to account for the early retirement discount. Conversely, if the individual retires late, her observed benefit will be greater than the calculated full retirement benefit.

C. *Adjusting life expectancy by income*

We adjust for differential life expectancy across income centiles using data from Chetty et al. (2014) as reported by the Health Inequality Project (HIP). These data provide life expectancy at age 40 for each lifetime income centile from 2001 to 2014. Since our sample starts in 1989 and goes until 2014, we apply the 2001 data for all years between 1989–2001 and the 2014 data for 2014–2019. Assigning the 2001 values to previous years seems to be a reasonable assumption, as the life expectancy differential between high and low income individuals is flat from 2001–2007, then expands after the 2008 Financial Crisis, as shown in Figure IA1.

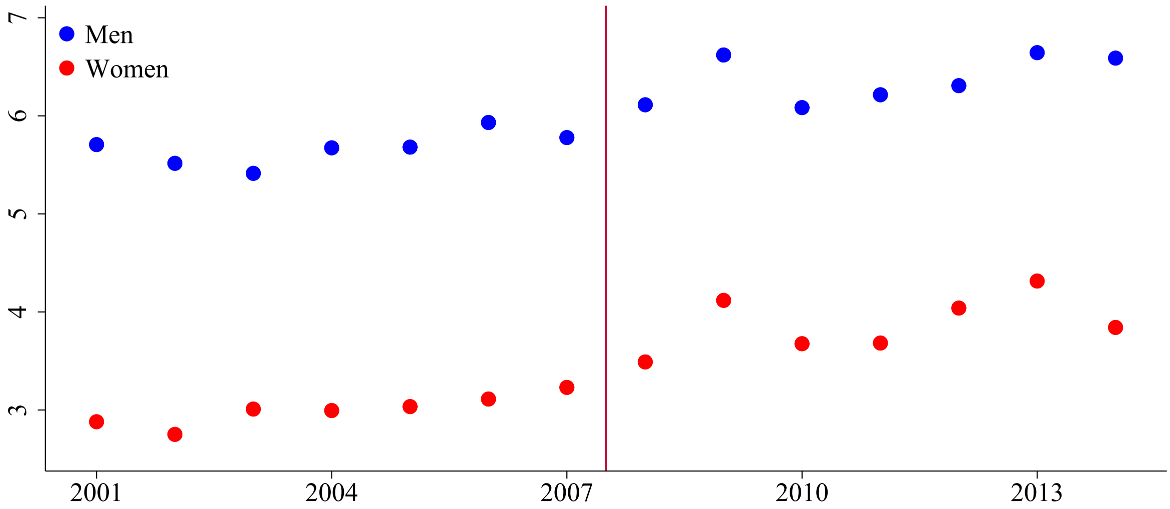
Using these data, we compute the number of years fewer (more) that a retired SCF respondent will live given their lifetime income centile. We then adjust the respondents age to reflect the shorter (longer) longevity implied by the data. To do this, the compute the *life expectancy spread* for each lifetime income centile in the HIP data, which is given by

$$\text{Life Expectancy Spread}_{centile,t} = \frac{\text{Life Expectancy}_{centile,t}}{\frac{1}{100}\sum_{centile=1}^{100} \text{Life Expectancy}_{centile,t}}.$$

We then take these life expectancy spreads and merge them with our primary mortality dataset coming from the Human Mortality Database (HMD). We then calculate the number of years fewer (more) people in the lower (higher) centiles of the income distribution live based on the

Figure IA1. Life expectancy differential, 2001–2014

This figure plots the difference in life expectancy for people in the top half and bottom half of the lifetime earnings distribution. The differences for men and women are plotted separately. The vertical line in the middle of the graph denotes the period before and after 2007.



unconditional life expectancy (i.e. at age 0). We define this as the *year difference* which is given by

$$\text{Year Difference}_{centile,t} = (\text{Life Expectancy Spread}_{centile,t} - 1) \times \text{Unconditional Life Expectancy}_t$$

which is rounded to the nearest integer. Note, that this will be negative for people in the bottom half of the lifetime income distribution and positive for people in the top half. From this, we calculate the *effective mortality age* for each SCF respondent, which is given by

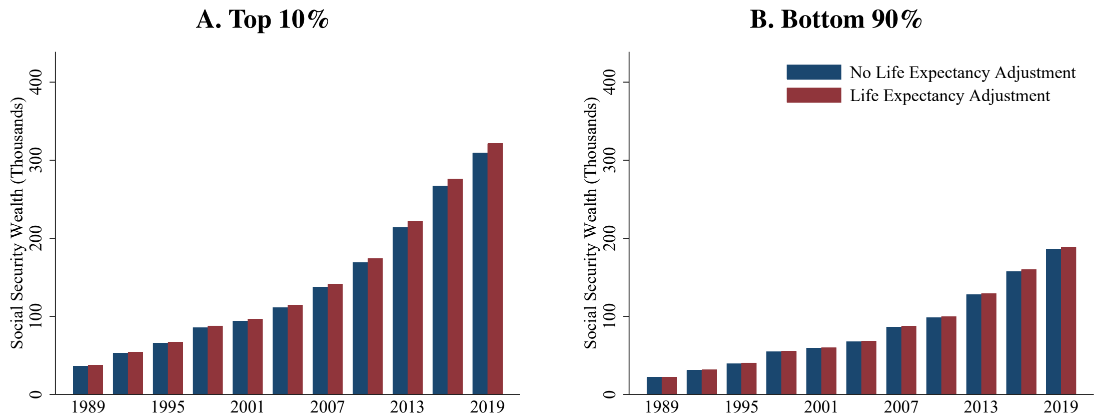
$$\text{Effective Mortality Age}_{i,centile,t} = \text{Current Age}_i - \text{Year Difference}_{centile,t}.$$

We then assign survival probabilities to that individual based on their effective mortality age.

Completing the life expectancy adjustment requires a valid proxy for lifetime income. Un-

Figure IA2. Adjusting for differential in life expectancy

This figure shows per capita Social Security wealth for each person in the SCF, applying population weights, for people in the top 10% (Panel A) and bottom 90% (Panel B) of the non-Social Security wealth distribution. Life expectancy adjusted values incorporate differential life expectancy across income centiles using data from the Health Inequality Project (HIP), as outlined in Appendix C.



fortunately, the SCF does not provide income histories. However, we can extrapolate based on the Social Security retirement benefits centile. Since Social Security benefits are a monotonically increasing function of lifetime income, this proxy allows us to preserve the order of individuals within the lifetime income distribution, which we then apply to the life expectancy adjustment.

An example is illustrative on this procedure: the life expectancy for men in 2019 in the HMD data is 76 years, and in that year, a person in the 1st lifetime income centile lives approximately 9 years less than the average person. Therefore, a 40 year old man in the 1st lifetime income centile has an effective mortality age of 49 years old, and he would be assigned the survival probabilities of a 49 year old man in 2019. We apply this life expectancy correction both to retired workers and to those still in the workforce, whose earnings histories we simulate.

When differences in mortality rates are accounted for, per capital Social Security wealth that accrues to the bottom decile falls by nearly 20%, and per capita Social Security wealth falls for the bottom six deciles. We modify our estimates of cohort Social Security wealth to reflect these differences.

However, this adjustment does not have a large impact on top wealth shares. This is due to an increase in the benefit-weighted average life expectancy of beneficiaries in the bottom 90% which can be seen in Figure IA2. Specifically, those in upper deciles of the marketable wealth distribution live for longer (more years of benefits) than those in lower deciles. Within the bottom 90%, the effect of this adjustment is to decrease benefit-years for individuals with lower benefits, and increase benefit-years for individuals with higher benefits.

As such, adjusting for the relationship between income level and mortality rates increases Social Security wealth for both the top and bottom of the overall wealth distribution. Though the increase in aggregate Social Security wealth goes disproportionately to the wealthy, it remains, nonetheless, much more equally distributed than marketable wealth.[‡]

D. *Capitalizing implied survivor benefits*

Widows can receive a share of the Social Security benefits of their deceased spouses. We account for this when capitalizing benefits by computing how likely it is that a respondent's spouse is alive given that the respondent is deceased, under the assumption that the survival probabilities of the couple are uncorrelated. In particular, widows can receive the maximum of their benefit and their deceased spouse's benefit. The implied present value of survivor benefits is therefore given by

$$\text{Implied Survivor Benefits}_{i,t} = \max\left\{\text{Spouse Benefits}_{i,t} - \text{Benefits}_{i,t}, 0\right\} \times \sum_{s=0}^{\infty} \frac{\prod_{k=t}^{s-1} m_{i,t+k}(1 - m_{i,t+k}^{spouse})}{1 + r_{t,t+s}}$$

where m represents the survival probability and r the real discount rate.

[‡]It is worth noting that this exercise illustrates the issue with a singular focus on top shares as a measure of wealth inequality. Differences in life expectancy disproportionately impact those at the bottom of the wealth distribution, but standard measures of wealth concentration focus on the share of aggregate wealth accruing to those at the top, thus missing out on such dynamics.

E. Proportion of people with no benefits

The vast majority of retirees receive some form of Social Security benefits. However, a fraction of retirees have insufficient work history to receive benefits. When aggregating Social Security benefits, we must take this into account. This requires a reasonable estimation of the proportion of people in each cohort that do not receive benefits.

We estimate this using Deaton and Paxson (1994) regressions for each gender, which is a constrained regression of the following form

$$\log(Pr(\text{No Retirement Benefits}))_{t,a,b} = \gamma_t + \eta_a + \delta_b + \varepsilon_{t,a,b} \tag{IA3}$$

subject to

$$\sum_{1989}^{2016} \gamma_t = 0 \tag{IA4}$$

$$\sum_{1989}^{2016} \gamma_t(t - 2002.5) = 0 \tag{IA5}$$

$$\eta_{72} = 0. \tag{IA6}$$

where a represents each age, t each survey year, and b each birth year.[§] The coefficients of interest are the birth year fixed effects, where this empirical set-up allows us to adjust for survey specific sampling error and age specific effects. The fitted values by birth year are shown in Appendix Figure IA6, where the average number of zero Social Security income respondents is shown to be 10% for men and 20% for women. In the simulation, these estimates are used to determining average Social Security wealth.

[§]Note that respondents are grouped into three-year age and birth year cohorts in this estimation.

F. Heterogeneous discount rates

This section gives greater detail on how the private discount rates from Section C are constructed.

Defining constrained households. This procedure identifies two different types of households: unconstrained and constrained. Unconstrained households are defined at those with no debt and greater than \$10,000 in 2019 dollars invested in liquid assets or greater than \$50,000 in 2019 dollars in quasi-liquid assets with each of these quantities defined by

$$\texttt{liquid\_wealth} = \texttt{liq} + \texttt{cds} + \texttt{nmmf} + \texttt{stocks} + \texttt{bond} + \texttt{savbnd}$$

and

$$\texttt{quasi\_liquid\_wealth} = \texttt{liquid\_wealth} + \texttt{retqliq} + \texttt{homeeq}.$$

The remaining households are defined as constrained. Since these cutoffs are somewhat arbitrary, we examine the results under alternative cutoffs. In particular, the results with more conservative of \$15,000 and \$75,000 in liquid and quasi-liquid wealth and more lax cutoffs of \$5,000 and \$25,000 in liquid and quasi-liquid wealth are within several basis points of the baseline results. This is because most households without debt in SCF have substantial liquid and quasi-liquid wealth.

Individual credit spreads. For constrained households, we construct a value-weighted interest rate for each individual in the SCF using the interest rates paid on mortgages and other property loans, auto and other vehicle loans, and personal loans. We then construct a spread over the safe rate by subtracting the annualized yield on the 5-year constant maturity treasury bond from Gürkaynak, Sack, and Wright (2008). An aggregate value-weighted spread is then constructed by combining households by their 5-year age group (e.g. 20–24 year olds, 25–29 year olds, etc.), survey year, and earnings quintile (constructed using the `income` variable in the SCF extract) level using the total amount of debt in each group and the SCF weights. Earnings quintiles are constructed within each 5-year age group and survey.

On credit card debt. In constructing individual credit spreads, we do not include credit card debt. There are two reasons for this. The first reason is motivated by the longstanding "credit card debt puzzle" in household finance (Gross and Souleles, 2002). Much of credit card debt that is rolled over month-to-month is held by households with substantial liquid assets. For example, among households rolling over a credit card debt in the 2019 SCF, the median total balance was $3,000 while median liquid wealth was exactly twice that, $6,000.[¶] Moreover, even when when paying off credit card debt, households do not pay down the card with the highest interest rate (Gathergood et al., 2019).

The literature around this puzzle has found that traditional channels like using credit cards to smooth income shocks or as short-term liquidity have relatively poor explanatory power (Laibson, Repetto, and Tobacman, 2003). Instead, various additional channels have been proposed to explain the puzzle like behavioral biases (Laibson, Repetto, and Tobacman, 2003), strategic motives around bankruptcy (Lehnert and Maki, 2002), within household bargaining (Bertaut, Haliassos, and Reiter, 2009) and spending shocks that cannot be remunerated with credit cards (Telyukova, 2013). Since many of these channels have little to do with the valuation of Social Security Wealth, we exclude credit card debt in this analysis.

The second reason is that, relative to Social Security wealth, credit card balances are quite small; the average balance is approximately $6,200 in the 2019 SCF. Presumably, households would not want to apply the very high average interest rate on credit cards (14%) to the entirety of their Social Security benefits, but rather only the portion that could fully reimburse the debt. It is, therefore, inappropriate to apply this rate to the entirety Social Security Wealth for

[¶]The median income for households rolling over credit card debt is also higher than that of household not rolling over credit card debt, though this may be driven by selection since very low income households are more likely to be ineligible for credit cards.

constrained households.

Estimating credit spreads and transition probabilities over the life-cycle. To understand how this spread evolves over the life-cycle, we estimate a Tobit model for each earnings quintile with the spread as the dependent variable and a cubic polynomial for age and year-fixed effects. The Tobit model prevents the fitted values from being lower than 0. We estimate the model on the average within each five-year age group, survey year, and earnings quintile. The model estimated is given by

$$s_{a,q,t} = \begin{cases} \gamma_{q,t} + \beta_{1,q}a + \beta_{2,q}a^2 + \beta_{3,q}a^3 + \varepsilon_{a,q,t} & \text{if } s^*_{a,q,t} > 0 \\ 0 & \text{if } s^*_{a,q,t} \leq 0 \end{cases}. \tag{IA7}$$

where $s^*_{a,q,t}$ is latent. We use the estimates of $s^*_{a,q,t}$ as the spread. This gives us the value of the spread conditional on having debt at each age and for each earnings quintile.

To understand how constrained households transition to unconstrained households over the life-cycle, we fit a Tobit model with a lower bound of 0 and upper bound of 1 for each year and earnings quintile, with the fraction of individuals receiving the market rate in each survey year-age-earnings quintile as the dependent variable and age as the independent variable. This is given by

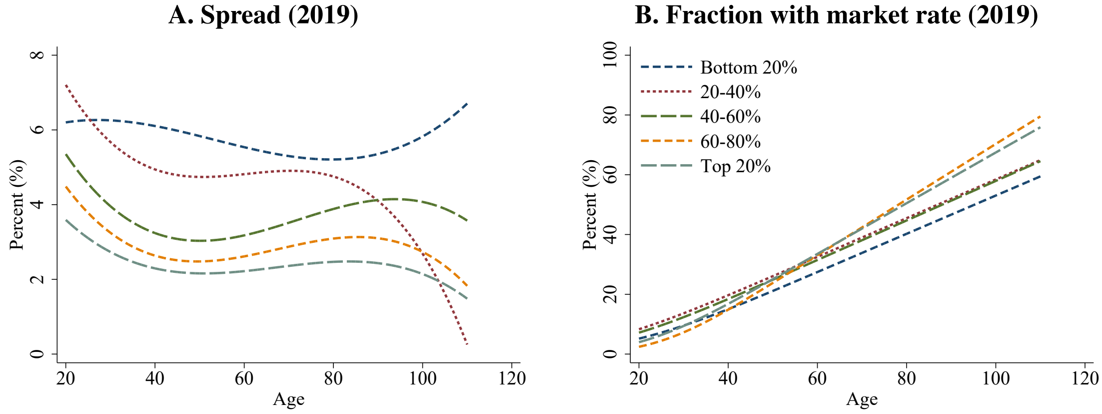
$$p_{a,q,t} = \begin{cases} 1 & \text{if } p^*_{a,q,t} \geq 1 \\ \zeta_{q,t} + \eta_{q,t}a + \epsilon_{a,q,t} & 0 < \text{if } p^*_{a,q,t} < 1 \\ 0 & \text{if } p^*_{a,q,t} \leq 0 \end{cases}. \tag{IA8}$$

where $p^*_{a,q,t}$ is latent. We use the estimates of $p^*_{a,q,t}$ as the transition probabilities.

Figure IA3 shows the discount rate adjustment and fraction discounting at the market rate for each earnings quintile across age in 2019. Panel A shows the estimated spread. When constructing forward rates for constrained households, we assign the $a + h - 1$ year spread for the h year forward rate. For example, a 30-year old receives the 30-year old spread added

Figure IA3. Discount rate adjustment and fraction receiving the market rate

Panel A shows the fitted values from a Tobit model from Equation (IA7) of the average value-weighted interest rate for each age and earnings quintile using the interest rates paid on mortgages and other property loans, auto and other vehicle loans, and personal loans less the annualized yield on the 5-year constant maturity treasury bond from Gürkaynak, Sack, and Wright (2008). The Tobit contains a cubic polynomial for age and year-fixed effects as the independent variables. Panel B shows the fitted values from a Tobit model from equation (IA8) of the fraction of households receiving the market discount rate with the independent variable is a linear term for age for each earnings quintile and survey year. This model is estimated separately for each year and earnings quintile. Values are restricted to be between 0 and 1.



to cashflows one year in the future, the 49-year old spread added to cashflows twenty years in the future, and so on. The discount rate adjustment has been rising through our sample and is lowest for the top 20% of earners. Panel B shows the fraction of households receiving the market discount rate at each age. This is rising throughout the life-cycle and across the income distribution.

Constrained yield curve. The predicted spreads and likelihood of transition are then used to construct the yield curve for constrained households in each age-earnings quintile-year. To do this, we make the assumption that households only transition from discounting according to the interest rate on their debt to the market discount rate, not the other way around. Under this assumption, the yield for constrained households can be expressed recursively as

$$\tilde{y}_{h,a,q,t} = \Bigg((1+\tilde{y}_{h-1,a,q,t})^{1-h} \frac{1}{1+f^{\text{unconstrained}}_{h,a,q,t}} - \left(\frac{1-p_{a+h-1,q,t}}{1-p_{a,q,t}}\right)(1+y^{\text{constrained}}_{h-1,a,q,t})^{1-h}\left(\frac{1}{1+f^{\text{unconstrained}}_{h,a,q,t}} - \frac{1}{1+f^{\text{constrained}}_{h,a,q,t}}\right)\Bigg)^{-\frac{1}{h}} - 1 \quad \text{(IA9)}$$

where $y^{\text{constrained}}$ is the constrained yields and the initial condition is $\tilde{y}_{0,a,q,t} = y^{\text{constrained}}_{0,a,q,t} = 0$. The forwards rates here are given by

$$f^{\text{unconstrained}}_{h,a,q,t} = f^{\text{risk-adj}}_{t,h}$$

and

$$f^{\text{constrained}}_{h,a,q,t} = \begin{cases} f^{\text{risk-adj}}_{t,h} & \text{with probability } p_{a+h-1,q,t} \\ f^{\text{risk-free}}_{t,h} + s_{a+h-1,q,t} & \text{with probability } 1-p_{a+h-1,q,t} \end{cases}.$$

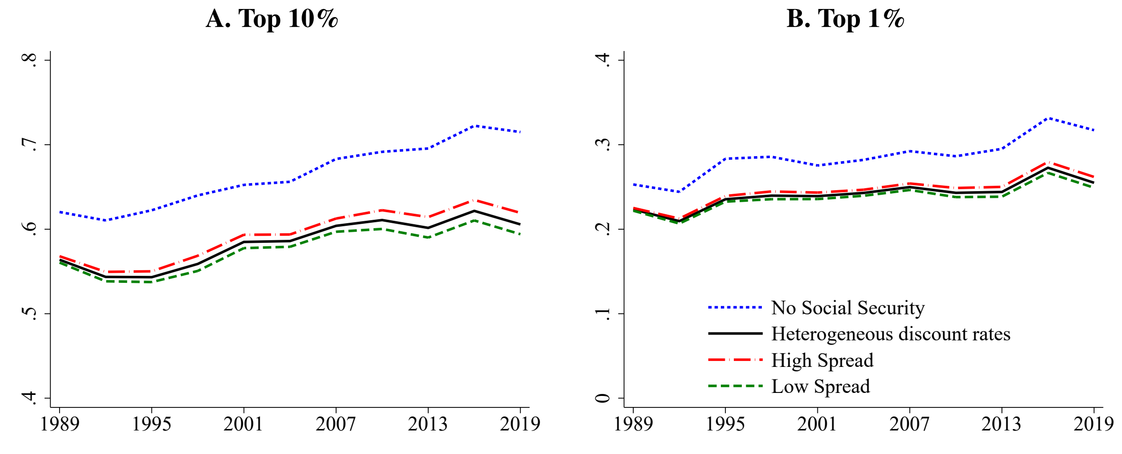
The resulting 10-year yields from this exercise are what is plotted in Panel 1 of Figure 10 for different earnings quintiles at each age in 1989 and 2019. The Panel 2 of Figure 10 combines the constrained and unconstrained yields to plot the average yield applied across both constrained and unconstrained households. This is given by

$$\bar{y}_{h,a,q,t} = \left(p_{a,q,t}(1+y^{\text{unconstrained}}_{h,a,q,t})^{-h} + (1-p_{a,q,t})(1+\tilde{y}_{h,a,q,t})^{-h} \right)^{-\frac{1}{h}} - 1. \quad \text{(IA10)}$$

Robustness. How sensitive are these results to either higher or lower discount rate spreads for constrained households? Figure IA4 presents the results multiplying and dividing the spreads by 1.5. Even when the spreads are increased by 50% the increase in wealth inequality is substantially offset. The top 10% share rises by 5.2 pp and top 1% share by 3.7 pp. This is approximately 60% of the rise in wealth inequality without Social Security in the SCF.

Figure IA4. Top 10% and Top 1% wealth shares — Higher and lower heterogeneous discount rates

This figure shows the top 10% and top 1% wealth shares under individuals private valuations of Social Security wealth with discount rate spreads multiplied and divided by 1.5.



C. Individual assignment procedure

This section discusses how we assign Social Security wealth to individuals for whom we do not observe data on Social Security benefits in the SCF. This involves assigning simulated Social Security wealth for non-recipients below age 66 and a backfilling methodology for non-recipients between 66 and 69.

Individuals below 66. For individuals below 66, we simulate future earnings paths to assign Social Security wealth. Specifically, we apply this procedure to all SCF respondents below the age of 62 and all respondents between the ages of 62 and 66 who have not yet claimed their Social Security benefits. The assignment method proceeds as follows:

1. We construct wage income for each individual in the SCF by splitting household `wageinc` between household members. To do this, we calculate the the fraction of reported individual wage income in the household accruing to the head of the household which is reported in the SCF raw data. We then multiple this fraction by `wageinc` from the SCF extract to obtain individual wage income. The reason we use this procedure, is that individuals often misreport their wage income in the raw SCF responses. Household

wage income, however, is more accurate as respondents are asked to report information from their tax filings, in particular, line 1 of IRS form 1040.

2. We construct an income matching variable for each individual in the SCF from the individual wage income variable from Step 1. This variable, which we will refer to as `match_inc`, is constructed by:

   (a) If income is less than five times the Social Security wage index in their survey year $w\bar{a}ge_t$, we round each individuals wage income to the nearest multiple of $.1 \times w\bar{a}ge_t$.

   (b) If income is greater than five times $w\bar{a}ge_t$ but less or equal to than twenty times $w\bar{a}ge_t$, we round each individuals wage income to the nearest multiple of $.5 \times w\bar{a}ge_t$.

   (c) If income is greater than twenty times $w\bar{a}ge_t$, we replace individuals wage income equal to $20 \times w\bar{a}ge_t$.

3. We construct an identical `match_inc` variable on current earnings for each simulated earnings path.

4. We create conditional expected Social Security wealth in the simulated data by averaging across simulated observations with the same year, age, gender, and `match_inc`.

5. We match each individual in the SCF with their respective year-age-gender-`match_inc` observation in the simulated data.

This method creates a match for all SCF respondents.

For individuals between 62 and 65, we need to make an additional adjustment. In particular,we set the assigned Social Security wealth to zero for some individuals to ensure that we have the correct portion of individuals receiving no benefits: 10% of men and 20% of women in each age-year-gender group. These are based on the results of the Deaton-Paxson estimation procedure shown in Appendix E. For these individuals, we reach the desired number of non-recipients for each year-age-gender group as follows:

1. We start with those assigned zero social security wealth by the simulation. If these individuals make up 10% (20%) of the population for men (women) in each age-year, we stop.

2. If not, we randomly assign zero social security wealth to individuals with the lowest income until we reach the 10% (20%) threshold for men (women).

Individuals between 66 and 69. For respondents aged 66–69, we do not simulate Social Security wealth and instead rely on a backfilling methodology for individuals not receiving Social Security benefits. The process for this is as follows:

1. We fill in the average observed retirement-adjusted Social Security wealth in the data for recipients aged 70–73 from the succeeding survey adjusted for inflation. For 2019, we fill in the average observed retirement-adjusted Social Security wealth in the data for recipients aged 70–73 from the 2019 survey.

2. We adjust these filled benefits downward, since these respondents also have a higher probability of being a non-recipient. This adjustment is given by

$$\text{adj}^{g}_{a,t,s} = \frac{\sum \mathbb{1}\{\text{No Benefits}\} - .1(1 + \mathbb{1}\{\text{Female}\})}{\sum \mathbb{1}\{\text{No Benefits}\}}$$

   where $\mathbb{1}\{x\}$ is an indicator variable equal to 1 when conditions x are met. This adjustment is calculated for each year-age-sex-population combination.

Individuals over 70. For all individuals older than 70, we obtain all values from the data. Nothing needs to be filled in for these observations, as there is no benefit from claiming after 70. In reality, some people may claim later, but we assume that these individuals will not receive benefits for the remainder of their lives.

D. Market beta of aggregate labor income

Consider the following exogenous system of stochastic processes

$$dy_t = -\kappa y_t dt + \begin{bmatrix} \sigma_l \\ -\sigma_s \end{bmatrix}^T dz_t, \tag{IA11}$$

$$ds_t = \left(\mu - \frac{\sigma^2}{2} + \phi y_t\right) dt + \begin{bmatrix} 0 \\ \sigma \end{bmatrix}^T dz_t, \tag{IA12}$$

$$l_{1,t} = y_t + s_t - \delta t, \tag{IA13}$$

$$d\pi_t = -r\pi_t dt - \begin{bmatrix} 0 \\ \lambda \end{bmatrix}^T \pi_t dz_t, \tag{IA14}$$

where y_t is log output, $s_t \equiv \log S_t$ is log stock price, $l_{1,t} \equiv \log L_{1,t}$ is log wage, π_t is the state-price density, $\lambda \equiv \frac{\mu - r}{\sigma}$, and $z_t = \begin{bmatrix} z_{1,t} \; z_{2,t} \end{bmatrix}^T$ is a standard Brownian motion. Note that, for now, we allow the $\sigma \neq \sigma_s$, which is different than in equation (12) and gives us a more general solution.

We want to find the beta at time t on a "wage strip", which is a security that pays out $L_{1,t+n}$ at $t + n$ and is denoted by

$$\beta_t^{L_1,n} = \frac{\text{Cov}_t\left(r_t^m dt, r_t^{L_1,n} dt\right)}{\text{Var}_t\left[r_t^m dt\right]}.$$

In this economy, the instantaneous return on the market r_t^m is defined by

$$r_t^m dt = \frac{dS_t}{S_t} = ds_t + \frac{1}{2}(ds_t)^2 = (\mu + \phi y_t) dt + \begin{bmatrix} 0 \\ \sigma \end{bmatrix}^T dz_t,$$

and the instantaneous return on the wage strip $r_t^{L_1,n}$ by

$$r_t^{L_1,n} dt = \frac{dP_t^{L_1,n}}{P_t^{L_1,n}},$$

where $P_t^{L_1,n}$ is the price of the wage strip. By no-arbitrage, the price of the wage strip is given by

$$P_t^{L_1,n} = \mathbb{E}_t \left[\frac{\pi_{t+n}}{\pi_t} L_{1,t+n} \right] = \mathbb{E}_t \left[\exp \left\{ \tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{1,t+n} \right\} \right], \tag{IA15}$$

where $\tilde{\pi}_t \equiv \log \pi_t$. The process $\tilde{\pi}_t$ is given by

$$\begin{aligned} d\tilde{\pi}_t &= \frac{d\pi_t}{\pi_t} - \frac{1}{2} \left(\frac{d\pi_t}{\pi_t} \right)^2 = \left(-r - \frac{1}{2}\lambda^2 \right) dt - \begin{bmatrix} 0 \\ \lambda \end{bmatrix}^T dz_t \\ \Rightarrow \tilde{\pi}_t &= \left(-r - \frac{1}{2}\lambda^2 \right) t - \begin{bmatrix} 0 \\ \lambda \end{bmatrix}^T z_t \end{aligned}$$

which comes from a straightforward application of Ito's lemma.

To solve equation (IA15), we are left with finding $l_{1,t+n}$, which is equivalent to solving for y_t and s_t. Using Ito's lemma, we find that

$$y_t = e^{-\kappa t} \left(y_0 + \begin{bmatrix} \sigma_l \\ -\sigma_s \end{bmatrix}^T \int_0^t e^{\kappa s} dz_s \right).$$

Now, to find s_t, we introduce a new variable $\tilde{s}_t$ defined as

$$\tilde{s}_t = s_t + \frac{\phi}{\kappa} y_t,$$

which is given by

$$d\tilde{s}_t = ds_t + \frac{\phi}{\kappa} dy_t = \left(\mu - \frac{\sigma^2}{2}\right) dt + \begin{bmatrix} \frac{\phi}{\kappa}\sigma_l \\ \sigma - \frac{\phi}{\kappa}\sigma_s \end{bmatrix}^T dz_t$$

$$\Rightarrow \tilde{s}_t = \left(\mu - \frac{\sigma^2}{2}\right) t + \begin{bmatrix} \frac{\phi}{\kappa}\sigma_l \\ \sigma - \frac{\phi}{\kappa}\sigma_s \end{bmatrix}^T z_t$$

Using this expression, we solve for s_t, yielding

$$s_t = \tilde{s}_t - \frac{\phi}{\kappa} y_t = \left(\mu - \frac{\sigma^2}{2}\right) t + \begin{bmatrix} \frac{\phi}{\kappa}\sigma_l \\ \sigma - \frac{\phi}{\kappa}\sigma_s \end{bmatrix}^T z_t - \frac{\phi}{\kappa} e^{-\kappa t} \left(y_0 + \begin{bmatrix} \sigma_l \\ -\sigma_s \end{bmatrix}^T \int_0^t e^{\kappa s} dz_s \right)$$

which implies that $l_{1,t}$ equals

$$l_{1,t} = y_t + s_t - \delta t = \left(\mu - \frac{\sigma^2}{2} - \delta\right) t + \begin{bmatrix} \frac{\phi}{\kappa}\sigma_l \\ \sigma - \frac{\phi}{\kappa}\sigma_s \end{bmatrix}^T z_t + \left(1 - \frac{\phi}{\kappa}\right) y_t.$$

Plugging everything back into the exponential expression of equation (IA15), we obtain

$$\begin{aligned}\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{1,t+n} &= \left(-r - \frac{1}{2}\lambda^2\right)(t+n) - \begin{bmatrix} 0 \\ \lambda \end{bmatrix}^T z_{t+n} - \left(-r - \frac{1}{2}\lambda^2\right) t + \begin{bmatrix} 0 \\ \lambda \end{bmatrix}^T z_t \\ &+ \left(\mu - \frac{\sigma^2}{2} - \delta\right)(t+n) + \begin{bmatrix} \frac{\phi}{\kappa}\sigma_l \\ \sigma - \frac{\phi}{\kappa}\sigma_s \end{bmatrix}^T z_{t+n} + \left(1 - \frac{\phi}{\kappa}\right) y_{t+n} \\ &= \left(-r - \frac{1}{2}\lambda^2\right) n + \left(\mu - \frac{\sigma^2}{2} - \delta\right)(t+n) + \begin{bmatrix} 0 \\ \lambda \end{bmatrix}^T z_t + \\ &\begin{bmatrix} \frac{\phi}{\kappa}\sigma_l \\ \sigma - \frac{\phi}{\kappa}\sigma_s - \lambda \end{bmatrix}^T z_{t+n} + \left(1 - \frac{\phi}{\kappa}\right) y_{t+n}\end{aligned}$$

Note that all components inside the exponent in equation (IA15) are normal variables. Hence, we can rewrite the equation as

$$P_t^{L_1,n} = \exp\left\{\mathbb{E}_t\left[\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{1,t+n}\right] + \frac{1}{2}\text{Var}_t\left[\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{1,t+n}\right]\right\}, \tag{IA16}$$

which leaves us with finding the two components in the exponent. Also note how we can express y_{t+n} via y_t:

$$\begin{aligned} y_{t+n} &= e^{-\kappa(t+n)}\left(y_0 + \begin{bmatrix} \sigma_l \\ -\sigma_s \end{bmatrix}^T \int_0^{t+n} e^{\kappa s} dz_s\right) \\ &= e^{-\kappa n}\left(y_t + \begin{bmatrix} \sigma_l \\ -\sigma_s \end{bmatrix}^T \int_t^{t+n} e^{\kappa(s-t)} dz_s\right)\end{aligned}$$

The first expression, $\mathbb{E}_t\left[\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{1,t+n}\right]$, is given by

$$\mathbb{E}_t\left[\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{1,t+n}\right] = \left(\mu - \frac{\sigma^2}{2} - \delta\right) t - \left(\frac{1}{2}\left(\lambda - \sigma\right)^2 + \delta\right) n + \left(1 - \frac{\phi}{\kappa}\right) e^{-\kappa n} y_t + \begin{bmatrix} \frac{\phi}{\kappa}\sigma_l \\ \sigma - \frac{\phi}{\kappa}\sigma_s \end{bmatrix}^T z_t$$

and the second expression, $\text{Var}_t\left[\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{1,t+n}\right]$, by

$$\begin{aligned}
\text{Var}_t\left[\tilde{\pi}_{t+n} - \tilde{\pi}_t + l_{1,t+n}\right] = &\left(\left(\frac{\phi}{\kappa}\sigma_l\right)^2 + \left(\sigma - \frac{\phi}{\kappa}\sigma_s - \lambda\right)^2\right) n \\
&+ \left(1 - \frac{\phi}{\kappa}\right)^2 \left(\sigma_l^2 + \sigma_s^2\right) \frac{1}{2\kappa}\left(1 - e^{-2\kappa n}\right) \\
&+ 2\left(1 - \frac{\phi}{\kappa}\right)\left(\frac{\phi}{\kappa}\sigma_l^2 + \frac{\phi}{\kappa}\sigma_s^2 - \sigma\sigma_s + \lambda\sigma_s\right) \frac{1}{\kappa}\left(1 - e^{-\kappa n}\right).
\end{aligned}$$

From this, we obtain the solution for $P_t^{L_1,n}$,

$$P_t^{L_1,n} = \exp\left\{at + b + cy_t + d^T z_t\right\}, \tag{IA17}$$

where

$$\begin{aligned}
a &\equiv \mu - \frac{\sigma^2}{2} - \delta \\
b(n) &\equiv -\left(\delta - \frac{1}{2}\frac{\phi^2}{\kappa^2}\left(\sigma_l^2 + \sigma_s^2\right) + \frac{\phi}{\kappa}\sigma_s\left(\sigma - \lambda\right)\right) n + \left(1 - \frac{\phi}{\kappa}\right)^2 \left(\sigma_l^2 + \sigma_s^2\right) \frac{1}{4\kappa}\left(1 - e^{-2\kappa n}\right) \\
&\quad + \left(1 - \frac{\phi}{\kappa}\right)\left(\frac{\phi}{\kappa}\left(\sigma_l^2 + \sigma_s^2\right) - \sigma_s\left(\sigma - \lambda\right)\right) \frac{1}{\kappa}\left(1 - e^{-\kappa n}\right) \\
c(n) &\equiv \left(1 - \frac{\phi}{\kappa}\right) e^{-\kappa n} \\
d &= \begin{bmatrix} \frac{\phi}{\kappa}\sigma_l \\ \sigma - \frac{\phi}{\kappa}\sigma_s \end{bmatrix}.
\end{aligned}$$

From equation (IA17), we can find the return on the wage strip by differentiating its price. To do that, we can rewrite its price as

$$P_t^{L_1,n} = \exp\left\{P_t^{L_1,n}\right\},$$

where

$$P_t^{L_1,n} = at + b(n) + c(n)y_t + d^T z_t.$$

By Ito's lemma we have (note that $dn = -dt$)

$$dP_t^{L_1,n} = \left(a - b'(n) - c'(n)y_t - \kappa c(n)y_t\right)dt + \left(c(n)\begin{bmatrix}\sigma_l \\ -\sigma_s\end{bmatrix} + d\right)^T dz_t, \qquad \text{(IA18)}$$

where

$$\begin{aligned}
b'(n) &= \frac{1}{2}\left(\sigma_l^2 + \sigma_s^2\right)\left(\frac{\phi}{\kappa} + c\right)^2 - \sigma_s\left(\sigma - \lambda\right)\left(\frac{\phi}{\kappa} + c\right) - \delta \\
c'(n) &= -\kappa\left(1 - \frac{\phi}{\kappa}\right)e^{-\kappa n} = -\kappa c(n).
\end{aligned}$$

Then, the return on the wage strip equals

$$\begin{aligned}
r_t^{L_1,n}dt &= \frac{dP_t^{L_1,n}}{P_t^{L_1,n}} = dP_t^{L_1,n} + \frac{1}{2}\left(dP_t^{L_1,n}\right)^2 \\
&= \left(a - b'(n) + \frac{1}{2}\left(c\sigma_l + \frac{\phi}{\kappa}\sigma_l\right)^2 + \frac{1}{2}\left(\sigma - \frac{\phi}{\kappa}\sigma_s - c\sigma_s\right)^2\right)dt + \begin{bmatrix}c\sigma_l + \frac{\phi}{\kappa}\sigma_l \\ \sigma - \frac{\phi}{\kappa}\sigma_s - c\sigma_s\end{bmatrix}^T dz_t
\end{aligned}$$

meaning that the expected return is

$$\mathbb{E}_t\left[r_t^{L_1,n}\right]=\mu-(\mu-r)\,\frac{\sigma_s}{\sigma}\left(\frac{\phi}{\kappa}+c\right).$$

This gives the beta on the wage strip as

$$\beta^{L_1,n}=\frac{\mathrm{Cov}_t\left(r_t^m dt, r_t^{L_1,n}dt\right)}{\mathrm{Var}_t\left[r_t^m dt\right]}=1-\frac{\sigma_s}{\sigma}\left(\frac{\phi}{\kappa}+c\right)$$

Further, we can test if the CAPM holds in this economy. To do this, we assess if $\mathbb{E}_t\left[r_t^{L_1,n}-r\right]=\beta^{L_1,n}\mathbb{E}_t\left[r_t^m-r\right]$ holds. The RHS of the expression is given by

$$\beta^{L_1,n}\mathbb{E}_t\left[r_t^m-r\right]=\left(1-\frac{\sigma_s}{\sigma}\left(\frac{\phi}{\kappa}+c\right)\right)\left(\mu-r+\phi y_t\right)$$

and the LHS by

$$\mathbb{E}_t\left[r_t^{L_1,n}-r\right]=\left(1-\frac{\sigma_s}{\sigma}\left(\frac{\phi}{\kappa}+c\right)\right)(\mu-r)\,.$$

Therefore, the CAPM only holds when y_t is zero in this economy.

Finally, note that if we assume no contemporaneous correlation between the labor and stock market ($\sigma_s=\sigma$), the results reduce to

$$\begin{aligned}\beta_t^{L_1,n}&=\left(1-\frac{\phi}{\kappa}\right)\left(1-e^{-\kappa n}\right)\\ \mathbb{E}_t\left[r_t^{L_1,n}\right]&=\left(1-\frac{\phi}{\kappa}\right)\left(1-e^{-\kappa n}\right)(\mu-r)+r\end{aligned}$$

while the discount rate remains unchanged as it does not depend on σ_s. So, when $n\to\infty$, the beta converges to $1-\frac{\phi}{\kappa}=1-\frac{0.08}{0.16}=0.5$.

E. Additional figures

Figure IA5. Market Implied and Social Security Administration Yield Curve Estimates

This figure presents the differences between the yield curves implied by treasury markets and those used in SSA reports. The market series is extended by extrapolating the 29-to-30 year forward rate into the future, as described in Section II.

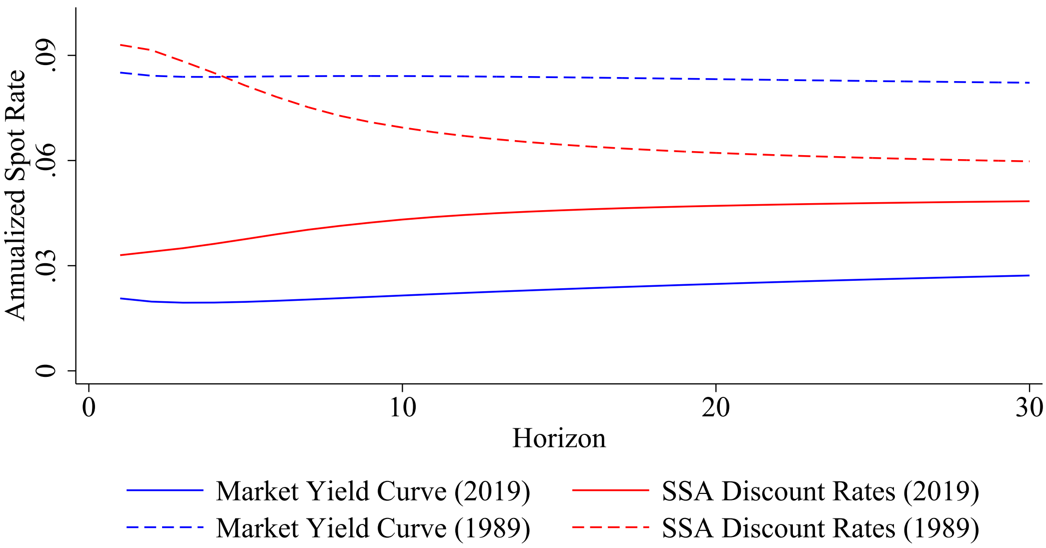


Figure IA6. Zero-Social Security Income Estimates: Deaton-Paxson Regressions

This figure shows the results for the Deaton-Paxson regressions outlined in Appendix E. The solid lines represent the estimated proportion of male and female respondents not receiving benefits after adjusting for survey-year and age specific fixed effects in a constrained. The dashed lines represent the mean proportion not receiving benefits for the 1929-1953 birth cohorts.

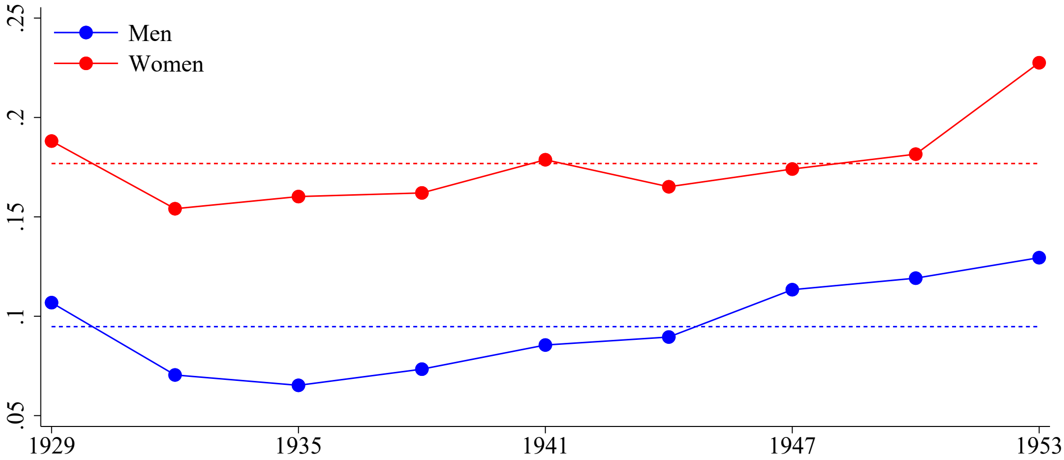


Table IAI. Calibration of labor income process

Parameter estimates for Sections III come from Specification (6) in Guvenen et al. (2021). Parameters can be found in Table IV and Table D.3 of the published version.

Parameter	Calibration
ρ	0.959
p_z	40.7%
$\mu_{\eta,1}$	-0.085
$\sigma_{\eta,1}$	0.364
$\sigma_{\eta,2}$	0.069
$\sigma_{z_1,0}$	0.714
λ	0.0001
p_ε	13.0%
$\mu_{\varepsilon,1}$	0.271
$\sigma_{\varepsilon,1}$	0.285
$\sigma_{\varepsilon,2}$	0.037
σ_α	0.300
$\sigma_\beta \cdot 10$	0.196
$\text{corr}_{\alpha\beta}$	0.768
$a_\nu \cdot 1$	-3.353
$b_\nu \cdot t$	-0.859
$c_\nu \cdot z_t$	-5.034
$d_\nu \cdot t \cdot z_t$	-2.895
$a_{z_1} \cdot 1$	0.407