Sovereign default and the decline in interest rates*

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Abstract

Sovereign debt yields have declined dramatically over the last half-century. Standard explanations, including aging populations and increases in asset demand from abroad, encounter difficulties when confronted with the full range of evidence. We propose an explanation based on a decline in inflation and default risk, which we argue is more consistent with the long-run nature of the interest-rate decline. We show that a model with investment, inventory storage, and sovereign default captures the decline in interest rates, the stability of equity valuation ratios, and the recent reduction in investment and output growth coinciding with the binding zero lower bound.

Keywords: Savings glut, Inflation expectations, Rare disasters, Secular stagnation
JEL codes: E31, E43, G12

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1 Introduction

Over the last four decades, interest rates across the developed world have starkly declined. Low interest rates, together with low output growth following the Great Recession of 2009, have evoked, for some, the possibility of "secular stagnation," a term coined by Hansen (1939) to describe a persistent period of low investment, employment, and growth. Summers (2015) and Gordon (2015) argue for the relevance of Hansen's concept from two angles: demand-side—an increase in demand for savings arising from changing demographics (Auclert et al., 2021, Eggertsson et al., 2019) or growing inequality (Auclert and Rognlie, 2017, Mian et al., 2021)—and supply-side—arising from a decline in the ideas and dynamism that have fueled the economic growth of the last half-century. A complementary idea is that of a "global savings glut" (Bernanke, 2005, Caballero et al., 2008): there is too great a supply of savings, mainly from patient investors outside the United States, compared to demand arising from the need to fund productive activities (ideas for which may be lacking).

But is a greater desire for savings, in fact, underlying the decline in interest rates? On some level, the link appears too obvious to be worth questioning. Yet any explanation based on a greater desire for savings runs into a significant problem when one also considers evidence from equity valuations and from capital investment. A decline in interest rates is, definitionally, equivalent to an increase in bond prices. A greater desire for savings should have raised stock prices to a similar degree as bond prices, but it did not. Likewise, a savings glut should have resulted in an investment boom, but the rate of capital investment has declined. From the point of view of the literature on increased desire for savings, low interest rates, and low growth, the behavior of equity valuations and firm investment is a puzzle. In response to this puzzle, Farhi and Gourio (2018) jointly consider growth, interest rates, and stock valuations in a neoclassical growth model that allows for rare disasters (Barro, 2009, Gourio, 2012). They argue that a substantial increase in the risk of rare disasters is necessary to jointly reconcile the level of interest rates and stock prices.

While Farhi and Gourio (2018) succeed in accounting for the joint behavior of stock prices and interest rates, an explanation based on increased fears of a disaster runs into its own problems. First, fears of rare disasters should be reflected in option prices, and specifically the VIX. While the VIX varies over time, its average level in the first and second half of our sample is almost exactly the same. Second, an explanation based on rare disasters is fragile because it also rests on assumptions about the elasticity of intertemporal substitution (EIS). Suppose that some underlying force were to be driving the risk of rare disasters upward. In an economy in which the EIS is above one, that force will drive valuations down; but if the EIS is less than one, it will cause valuations to rise, deepening the puzzle. Moreover, in a production model, rising disaster risk and an EIS below one counterfactually imply rising investment and growth. One could reverse the directionality, assuming a *decrease* in the risk of rare disasters, but this simply highlights the fact that there is no reason in the first place to believe disaster fears have increased.

We therefore propose a different explanation, one based on a decline in the risk of sovereign default. Greater trust in the sovereign's ability and willingness to repay debts could well have driven the decline in interest rates spanning centuries (Ferguson, 2018). Most recently, it is likely that reduced risk of default manifests through reduced inflation expectations. Indeed, there is substantial evidence for a steady decline in inflation expectations, spanning the 30 years over which interest rates have declined. Evidence from options markets suggests that inflation expectations became "anchored" in the twenty-first century—that is, investors did not fear either very high or very low inflation (Reis, 2020). When one takes this evidence into account, it is not difficult to jointly explain the decline in interest rates and the stability of stock valuation ratios. Because the true real rate has not declined as much, valuation ratios rise less, and there is no need to assume a large increase in the probability of a rare disaster to explain the evidence.

One may wonder: if it is simply inflation expectations that have declined, why is it that *measured* real rates, namely nominal rates minus ex post realized inflation, have also declined? This apparent disconnect disappears if one accounts for inflation *risk*, the risk that the price level rises during recessions or depressions. Indeed, if inflation were perfectly forecastable, then a change in inflation expectations should not impact ex post real rates. However, inflation, or the lack thereof, can come as a surprise. A decline in inflation risk will lead investors to require a lower premium to hold nominal securities. Interest rates will decline if this premium declines, even if measured in real terms ex post. This effect is more pronounced if investors fear inflation that, in sample, does not occur. From the point of view of cash flows, and given that the sovereign has control over the money supply, inflation risk is essentially risk of default (Barro, 2006). A decline in inflation risk is thus a decline in the probability of default, and may even affect rates on securities that are said to be inflation-protected. Our first contribution is to show that a model with rare disasters and a decline in inflation risk can explain the decline in interest rates and the stability of valuations.

We find direct support for declining inflation risk in the data. First, we compare returns on nominal and inflation-indexed bonds. Absent an inflation risk premium, average real returns on nominal and inflation-indexed bonds should be identical. However, we show that, in the 1980s and 1990s, inflation-adjusted returns on U.K. nominal bonds were almost twice as high as the corresponding returns on inflation-linked bonds. This large premium disappeared in the twenty-first century, as our model predicts. Second, we document evidence of declining inflation risk from survey expectations and from the changing correlation between inflation and growth. This latter evidence is consistent with recent findings that the once-positive correlations between inflation and the output gap (Campbell et al., 2020) and bond and stock returns (Campbell et al., 2017) became negative in the 2000s. Finally, because sovereign risk depends on institutions that have altered substantially over the centuries, this explanation could account for the striking fact that current rates are low, not just relative to the last 40 years, but to the last 400 years (Schmelzing, 2020).

We therefore first account for the joint behavior of interest rates, inflation expectations, and stock prices assuming an endowment economy. However, there is more in the puzzle than an endowment framework alone can address. Just as the joint behavior of equities and interest rates constitutes a puzzle, so too does the behavior of investment and interest rates. The same increased desire to save should have led to an investment boom, and yet, in the data, investment declined, potentially contributing to the decline in growth. An endowment economy cannot speak to the determinants of investment and growth. We therefore broaden our inquiry by nesting our mechanism within a neoclassical growth model. We realistically allow for the existence of cash in such a model, implying a zero lower bound: if agents can always costlessly transfer wealth across periods, there is no need to pay to do so with a negative nominal interest rate.[1] When expected inflation and risk of inflation are low, cash emerges as an inventory technology, which rivals productive capital investment. Our second contribution is to show that low default risk amplifies forces lowering real interest rates such as increased patience and lower productivity, leading to yet lower growth and lower interest rates than otherwise. Our model therefore formalizes a notion of a deflationary trap that quantitatively accounts for the data.

While cash is one interpretation of money, so is sovereign debt, provided it is riskless (Reis, 2022). The role of government taxing and spending as a riskfree means of transferring resources over time is, for example, at the center of the analysis of Blanchard (2019).[2] When real interest rates are above the zero lower bound, or when default risk is present, cash in all its forms (including debt) is in zero net supply. At the zero lower bound, government debt and cash are in positive net supply, and are part of the wealth calculation in the economy. The government has an effective monopoly on the ability to create an inventory asset, similarly though not identically to the idea that government bonds could create liquidity services

[1]The theory of cash as inventory dates to Baumol (1952), who applies an inventory control analysis to the theory of money. Money as inventory also accords with the "social contrivance of money," as proposed by Samuelson (1958), which asserts that money can be used to obtain the socially optimal allocation in an overlapping generations framework in which the storage of consumption goods is impossible.

[2]Blanchard (2019) argues that this kind of intertemporal resource transfer is optimal when the riskfree rate falls below the growth rate of the economy ($r^* < g$). We argue that it becomes optimal through inventory at the zero lower bound ($r^* < 0$). Blanchard sets $g = 0$, so the intuition in his paper is identical.

(Barro, 1974). As further support for our analysis, we show that broad measures of cash have increased during the period in question. We emphasize that, though our model accounts for an increase in cash, it does not rely on any assumptions regarding the need for liquidity services or for a special role for safe assets.

The remainder of this paper is organized as follows. In Section 2, we briefly summarize the empirical evidence. Section 3 considers the ability of an endowment economy to match this evidence, either with changes in the probability of disaster, or changes in the probability of default. In Section 4, we solve the model with an inventory technology and show its implications for investment and growth. Section 5 concludes.

2 Summary of the data

Panel A of Figure 1 shows nominal government rates in a seven-century-long dataset collected by Schmelzing (2020). Interest rates are highly volatile, as Jordà et al. (2019) emphasize.[3] Periods of extreme spikes, and also low rates, occurred around the American Revolution, Napoleonic Wars, and World War II, reflecting a tension between an increase in risk of sovereign default and precautionary savings around disasters. High rates in the 1970s and 1980s clearly stand out. Nonetheless, the figure shows a steady decline. Perhaps a more dramatic demonstration comes from Figure 1, Panel B, which shows the Bank of England lending rate, from the start of when the series was available. Only in the very most recent period did this rate reach a zero lower bound.

Figure 2 narrows in on the last forty years, the focus of much of the literature. The federal funds rate in the U.S. declined sharply from 10% to 2% (Panel A).[4] On the other

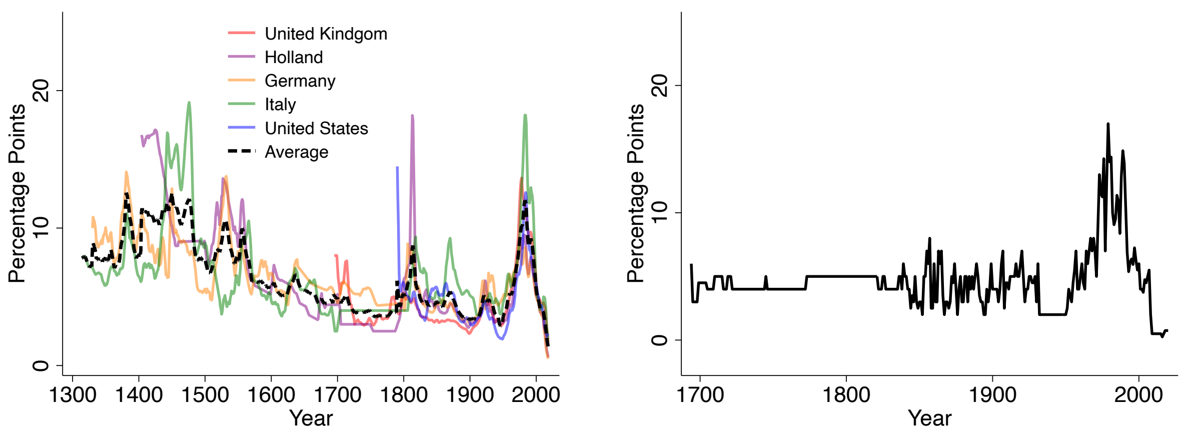
[3] Jordà et al. (2019) note that prior observations of a real rate of zero are not unusual. However, these are observations after subtracting ex post realized inflation, not ex ante inflation-adjusted yields. While it is true that both returns are zero from an investor's perspective, one was a realization of zero because of high inflation, whereas the other is an expected value of zero.

[4] In our quantitative model, we will focus on the one-year nominal yield. Notably, the literature studying monetary policy shocks and real interest rates finds that the secular decline in short-term rates has also

Figure 1: Nominal government rates

Panel A shows a five-year moving average of long-term nominal sovereign yields in the United Kingdom, Holland, Germany, Italy, and the United States from 1311–2018. The solid black line represents an average of all of the plotted series. Yields are from Schmelzing (2020) and are in annual terms. Yields come from a variety of archival, primary, and secondary sources. Panel B shows the nominal lending rate for the Bank of England expressed in annual terms.

A. Long-term sovereign yields **B. Bank of England lending rate**



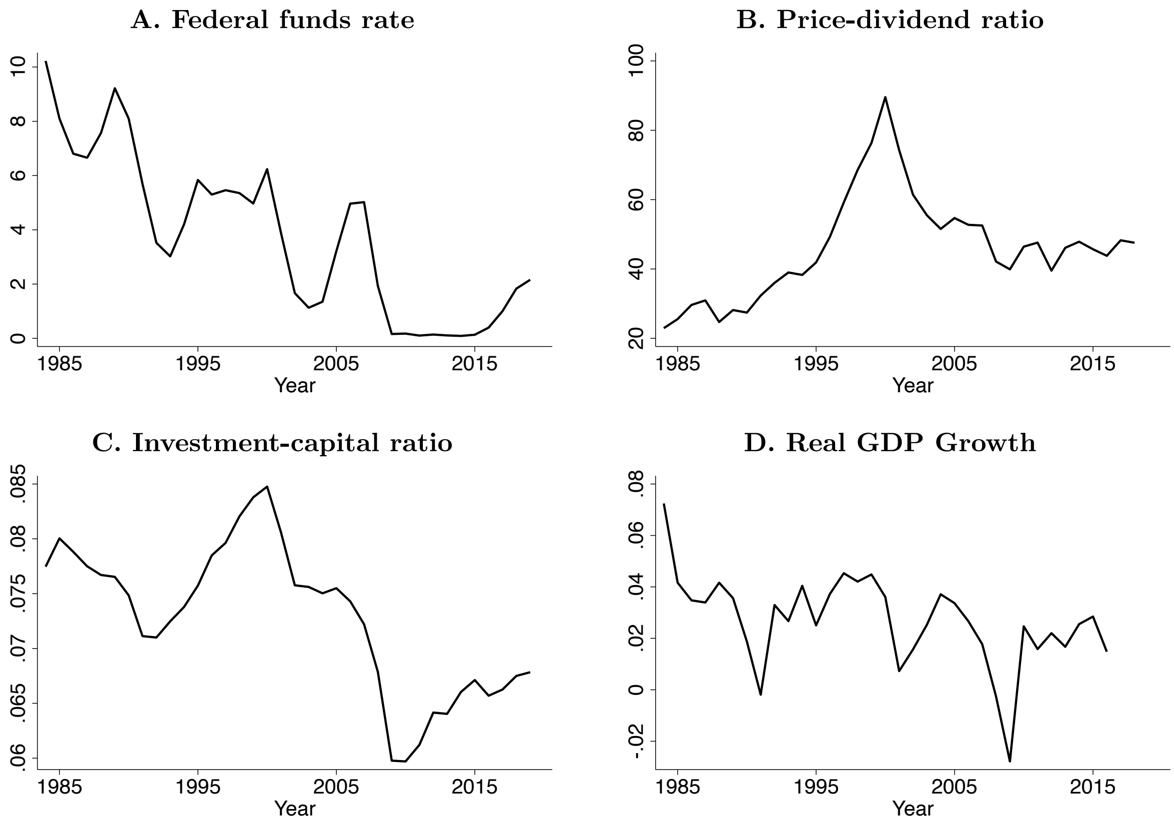
hand, the price-dividend ratio has gone from around 20 to 50, implying a dividend yield of approximately 5% going to 2%—a smaller decline (Panel B). Moreover, the last row of Figure 2 displays the decline in the investment-capital ratio (Panel C) and real GDP growth (Panel D) since the 1980s. Investment as a percentage of the capital stock went from an average of 7.7% to 6.9%, while real GDP growth declined from an average of 3.7% to 1.9%.

Figure 3 shows a longer time series of the price-dividend ratio, and also includes the cyclically-adjusted price-earnings (CAPE) ratio and the price-dividend ratio from the United Kingdom. It shows that the price-dividend ratio shifted upward in the late 1990s. This pattern does not appear in the CAPE ratio, nor in the U.K., and therefore may reflect a use of repurchases rather than cash payments as a means of returning cash to shareholders, and not a decline in interest rates (Boudoukh et al., 2007, Fama and French, 2001). For more

shown up in long-term yields (Bianchi et al., 2022, Hillenbrand, 2022). For example, the 30-year yield fell by the same amount since the mid-1980s as the federal funds rate.

Figure 2: Various data moments, United States from 1984–2018

The figure shows the effective federal funds rate (shown in annual percentage points), the annual price-dividend ratio for the United States on the value-weighted CRSP index, the investment-capital ratio, and the annual real GDP growth rate for the United States.



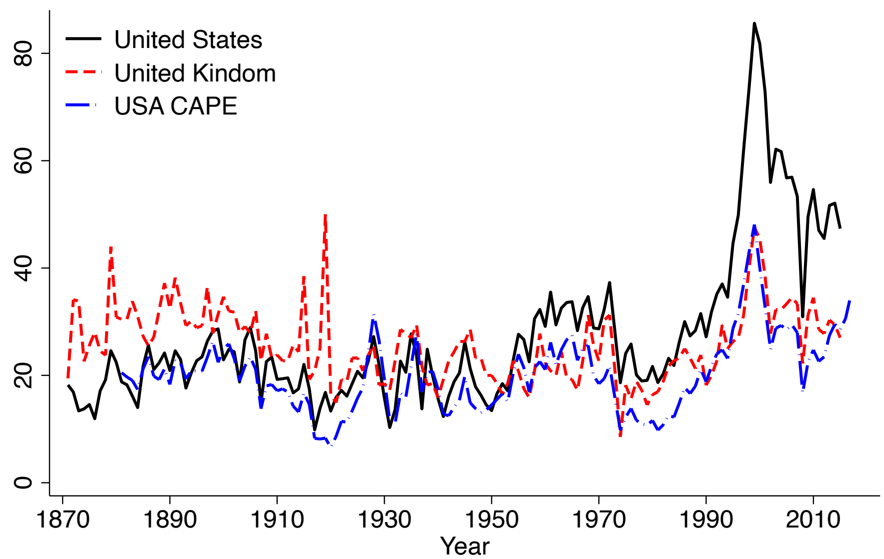
information on the data and sources, see Appendix A.

3 Endowment economy

We first turn to a standard endowment economy with a representative agent. To interpret the secular decline in interest rates, we calibrate the model separately to two sample periods, 1984–2000 and 2001–2021. We identify the year of this structural break (2001) by conducting

Figure 3: Price-dividend and price-earning ratios: United States and United Kingdom

The figure shows the price-dividend ratio for the United States and United Kingdom since 1870 and the U.S. cyclically-adjusted price-earnings (CAPE) ratio. The black, solid line shows data for the United States price-dividend ratio and the red, solid line shows data for the price-dividend ratio of the United Kingdom. Price-dividend ratios are the end-of-year price divided by the sum of all dividends from the preceding year. The blue dashed-dotted line shows the CAPE ratio.



a break test on the time series of one-year inflation-adjusted Treasury bill yields. Details can be found in Appendix B. This is the same breakpoint used by Farhi and Gourio (2018), who perform a similar analysis; it is also consistent with evidence from Campbell et al. (2020), who find a structural break in the relationship between GDP growth and inflation in 2001. While this approach of comparing sample averages means that certain features of the data (such as high-frequency volatility of prices and interest rates) remain outside the scope of the analysis, it allows us to consider the possibility of long-run unforeseen structural changes. Farhi and Gourio assume a neoclassical growth model. We will return to such a model in the next section, but for the analysis at hand the extra degree of complication is not necessary. As far as prices and interest rates are concerned, and in this i.i.d.-growth-rate setting, the production model and the endowment model yield the same predictions.

The aggregate endowment evolves according to

$$C_{t+1} = C_t e^{\mu}(1 - \chi_{t+1}), \tag{1}$$

where χ_{t+1} represents an occurrence of rare disaster:

$$\chi_{t+1} = \begin{cases} 0 & \text{with probability} \quad 1-p \\ \eta & \text{with probability} \quad p, \end{cases} \tag{2}$$

for $\eta \in (0,1)$. Note that p represents the probability of a disaster and η its magnitude. We assume the representative agent has Epstein-Zin-Weil recursive preferences (Epstein and Zin, 1989, Weil, 1990) with risk aversion γ, elasticity of intertemporal substitution (EIS) ψ, and discount factor β. Let W_t denote the representative agent's wealth, here assumed to be the cum-dividend value of the consumption claim. Let $R_{W,t+1} \equiv W_{t+1}/(W_t - C_t)$ denote the return on wealth from time t to $t+1$. The stochastic discount factor (SDF) then equals

$$M_{t+1} \equiv \beta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1}, \tag{3}$$

for $\theta \equiv (1-\gamma)/(1-1/\psi)$.

In this section, we assume that the aggregate stock market equals aggregate wealth (ex-dividend) and that the ex post real return on the Treasury bill equals the riskfree rate. We relax these assumptions in the sections that follow. In equilibrium, $R_{W,t+1}$ must satisfy:

$$\mathbb{E}_t \left[M_{t+1} R_{W,t+1} \right] = 1. \tag{4}$$

Our assumptions and the endowment and preferences imply a constant price-dividend ratio $(W_t - C_t)/C_t$, which we denote by κ. Standard arguments (see Appendix C) then imply that

$$\kappa = \frac{\beta e^{(1-\frac{1}{\psi})\mu}\left[1+p((1-\eta)^{1-\gamma}-1)\right]^{\frac{1}{\theta}}}{1-\beta e^{(1-\frac{1}{\psi})\mu}\left[1+p((1-\eta)^{1-\gamma}-1)\right]^{\frac{1}{\theta}}}. \quad (5)$$

Given the return on the wealth portfolio, the Euler equation provides the return on the one-period riskless bond:

$$\log R_f = -\log\beta + \frac{1}{\psi}\mu - \log(1+p((1-\eta)^{-\gamma}-1)) + \left(\frac{\theta-1}{\theta}\right)\log(1+p((1-\eta)^{1-\gamma}-1)). \quad (6)$$

Equations (5) and (6) constitute a system of two equations in two unknowns, p and β. Combining (5) and (6) gives the equity premium:

$$\begin{aligned}\log \mathbb{E}_t[R_{W,t+1}] - \log R_f &= \log(1-p\eta) + \log(1+p((1-\eta)^{-\gamma}-1)) \\ &\quad - \log(1+p((1-\eta)^{1-\gamma}-1)) \\ &\approx p\eta((1-\eta)^{-\gamma}-1)\end{aligned}$$

where the approximation is accurate for small p.

3.1 Increasing disaster probability

We calibrate this model using measured growth rates of real per capital consumption $\mu = 0.0257$ from 1984 to 2000 and $\mu = 0.0148$ from 2001 to 2021.[5] For comparability with Farhi and Gourio, we first show results for their calibration, corresponding to $\gamma = 12$, $\psi = 2$, and a disaster size $\eta = 0.15$. We find similar results, in that we match the data using a

[5]This is obtained from series A794RX0Q048SBEA from FRED Economic Data hosted by the St. Louis Federal Reserve.

discount factor (β) of 0.969 in the early period and 0.982 in the later period, and a disaster probability (p) of 2.25% in the early period and 4.50% in the later period. We thus arrive at our first result: matching the combined stability of valuations and the decrease in riskfree rates requires a large increase in the disaster probability, even after accounting for decreased growth.[6]

It may seems surprising that we require such a large increase in p. After all, the interest rate did fall due to decreased growth and increased patience. Moreover, a decrease in growth moderates the interest rate effect on stock prices, leading to a price-dividend ratio lower than it would have been, which is precisely the problem we are trying to solve. As it happens, the reason we require such a large increase is that, like the growth rate, the increase in disaster probability has two offsetting effects which are cancelled out when the elasticity of intertemporal substitution (EIS) equals unity. The EIS acts as a free parameter in this explanation. While there is no fundamental reason to believe that the EIS should differ greatly from the inverse of risk aversion (the former governs the desire of the agent to smooth across time, the latter to smooth across states), Farhi and Gourio (2018) follow others in the literature in assuming a high value of risk aversion and a high EIS in order to match the equity premium without other counterfactual implications. For an increase in disaster risk to be the explanation, not only must the change be large enough to overcome the offsetting effects, but it is essential to assume that the EIS is above unity.

To illustrate this point, Panel C of Table 1 sets the EIS to 1/2 rather than 2, while keeping everything else the same. Lower growth and a rising disaster probability cause valuations to increase, not decrease. Matching (5) and (6) with p and β still requires an increase in p in the second period. However, β must now decline, implying that investors would need to have become less patient, not more, contradicting the demand-side intuition for the decline

[6] van Binsbergen (2020) states the puzzle as follows: given the decrease in interest rates and the duration of the stock market, one would have expected a much larger capital gain if the risk premium were to remain constant.

Table 1: Accounting for the data with a change in disaster probability

This table shows parameters necessary to match the data, assuming an endowment economy with rare disasters and no inflation risk. Unless otherwise noted, we take average consumption growth from the data, and calibrate the disaster probability p and the subjective discount factor β to fit average interest rates and the price-dividend ratios in each of two sample periods. Because there is no inflation or inventory storage in the model, the riskfree rate proxies for the ex post real yield on the Treasury bill (Treasury bill yield minus realized inflation, or "inflation-adjusted Treasury yield"), and the wealth-consumption ratio proxies for the price-dividend ratio on the aggregate market. The table shows how p and β change depending on assumptions regarding elasticity of intertemporal substitution (EIS) and on growth. Treasury yields in the data, and parameters in the model, are annual.

		Values	
	Parameter	1984–2000	2001–2016
Panel A: Moments in the data			
Price-dividend ratio	κ	42.34	50.11
Inflation-adjusted Treasury yield	y_b	0.0279	-0.0035
Panel B: $\gamma = 12$, EIS $= 2$, $\eta = 0.15$			
Average consumption growth	μ	0.0257	0.0148
Discount factor	β	0.969	0.982
Probability of disaster	p	0.0225	0.0450
Panel C: $\gamma = 12$, EIS $= 0.5$, $\eta = 0.15$			
Average consumption growth	μ	0.0257	0.0148
Discount factor	β	0.993	0.977
Probability of disaster	p	0.0225	0.0450

in interest rates (Summers, 2015).

3.2 Did the equity premium rise?

We now ask whether the equity premium did in fact rise. The literature studying long-run variation in the equity premium generally comes to the conclusion that the equity premium has declined over the postwar period, including from the first to the second periods that are our focus (Avdis and Wachter, 2017, Fama and French, 2002, Lettau et al., 2008, van Binsbergen and Koijen, 2010, 2011). This evidence contradicts a rise in disaster risk.

Options markets are another place to look for evidence of an increase in the equity premium (Barro and Liao, 2021).[7] Virtually any explanation for an increase in the ex ante equity premium involves an increase in risk or risk aversion. While it is possible that such risk is not realized in sample, option prices incorporate the probability market participants assign to such risk materializing. Figure 4 shows the VIX, reported by the Chicago Board Options Exchange (CBOE). The VIX is the risk-neutral expectation of quadratic volatility, which is tightly tied to the equity premium. While the VIX is highly volatile at high frequencies, the average level of the VIX is remarkably stable between the two periods: equal to 21 in both. It is hard to reconcile this stability with a secular increase in the equity premium.

Given a model, one can say more. In Appendix D, we show how to go from the endowment economy model to a value of the VIX. A higher disaster probability implies a significantly higher VIX, not only because the ex ante volatility is higher (due to possible disasters), but because the risk-neutral volatility is higher still. If we ask the model to explain the level of the VIX in the earlier sample, and then modify the disaster probability as required, the VIX would counterfactually rise from 21 to 23, rather than remain at 21 as in the data.[8] A test of whether the higher value is consistent with the data is rejected at the 1% level.

3.3 Sovereign default risk

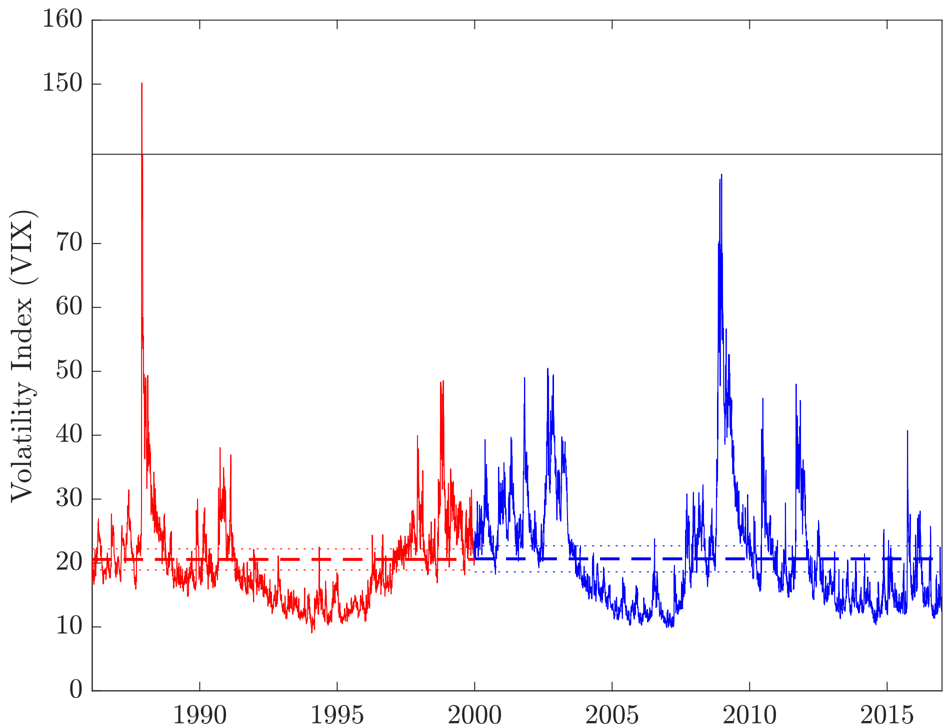
The typical empirical estimate of the equilibrium riskfree rate is the real return on government debt; however, this return is not necessarily riskless, as the government can default either outright or through inflation. We now price this claim by including partial default

[7] According to our argument, one cannot measure the risk premium from the difference between equity and bond returns, because the decline in the inflation risk premium on bonds will falsely suggest a rising excess return. Our evidence from options markets is immune to this concern.

[8] In a similar way, Siriwardane (2015) and Seo and Wachter (2018) back out measures of disaster risk using options data and do not find an increase in the probability of disaster over this period.

Figure 4: Chicago Board Options Exchange Volatility Index (VIX)

The figure plots the VIX series from 1986 to 2020 from the Chicago Board Options Exchange (CBOE). The long dashed red line is the average VIX from the beginning of the series to the end of the year 2000. The long dashed blue line shows the average VIX since the beginning of 2001. Estimated averages in both samples are plotted with a two-standard-error confidence interval where standard errors are adjusted for heteroskedasticity and autocorrelation (Newey and West, 1987) with two lags on the monthly VIX.



that co-occurs with disasters.[9] A decline in default risk can explain the secular trends in riskfree rates and valuation ratios since 1980 without appealing to rising disaster risk.

Suppose, in a disaster, creditors lose a fraction $\lambda\eta$ relative to the face value of the bond. That is, a bond issued at time t pays $1-L_{t+1}$ at time $t+1$, where loss $L_{t+1} = \lambda\chi_{t+1}$ represents a loss of zero if there is no disaster, and $\lambda\eta$ if a disaster should occur. If $\lambda = 1$, the loss to creditors is equal, in percentage terms, to the decline in consumption η. If $\lambda = 0$, then the

[9]See Appendix C.3 for more detailed derivations.

bond is riskfree. Let Q_t be the price of the defaultable bond. In equilibrium,

$$Q_t = \mathbb{E}_t \left[M_{t+1}(1 - L_{t+1}) \right]. \tag{7}$$

Let $y_{b,t}$ denote the continuously-compounded yield on this bond. That is, $y_{b,t} \equiv -\log Q_t$. Because the yield is constant, we will simply refer to this quantity as y_b. Note that the yield equals the return in the case of no default. Evaluating (7) implies:

$$y_b = \log R_f + \log\left(1 + p((1-\eta)^{-\gamma} - 1)\right) - \log\left(1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right) \approx \log R_f + p\lambda\eta(1-\eta)^{-\gamma} \tag{8}$$

where R_f is the gross riskfree rate from (6). For $\lambda > 0$, the yield exceeds the riskfree rate. Letting $R_{b,t+1} \equiv (1 - L_{t+1})/Q_t$ denote the return on the defaultable bond, the expected return is

$$\begin{aligned}
\log \mathbb{E}[R_{b,t+1}] &= \log R_f + \log(1 - p\lambda\eta) + \log\left(1 + p((1-\eta)^{-\gamma} - 1)\right) \\
&\qquad - \log\left(1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right) \\
&\approx \log R_f + p\lambda\eta((1-\eta)^{-\gamma} - 1).
\end{aligned} \tag{9}$$

The term $p\lambda\eta((1-\eta)^{-\gamma} - 1)$ is the default risk premium. Notice that the yield y_b equals

$$y_b = \log \mathbb{E}[R_{b,t+1}] - \log(1 - p\lambda\eta) \approx \log \mathbb{E}[R_{b,t+1}] + p\lambda\eta, \tag{10}$$

and exceeds both the riskfree rate and the expected return on the bond when $\lambda > 0$. In a sample in which no disasters occur, the average ex post real return on the bond will correspond to the yield (8), not the expected return (9).

We have thus far been agnostic as to the means of default. Inflation offers one such means. To make the connection precise, let Π_t denote the price level and $\Delta\pi_t = \log(\Pi_{t+1}/\Pi_t)$

inflation. A capital loss of L_{t+1} through default is equivalent to an inflation of $1/(1 - L_{t+1})$. We also allow for inflation to occur outside of disasters. We will assume for notational convenience that all such inflation is known one period in advance (that is, it is locally deterministic), but that is not important. To summarize, the price level follows the process

$$\Pi_{t+1} = \Pi_t e^{\mu_{\pi,t}}(1 - L_{t+1})^{-1}, \tag{11}$$

where $\mu_{\pi,t}$ represents growth in the price level that is locally deterministic. Now consider the nominal price on the nominal bond, denoted $Q_t^\$$. In equilibrium,

$$Q_t^\$ = \mathbb{E}_t\left[M_{t+1}\frac{\Pi_t}{\Pi_{t+1}}\right]. \tag{12}$$

Let $y_{b,t}^\$$ denote the continuously-compounded yield, namely $y_{b,t}^\$ \equiv -\log Q_t^\$$. Finally, let $R_{b,t+1}^\$ \equiv 1/Q_t^\$$ denote the nominal return on the nominal bond. It is also the case that $R_{b,t+1}^\$ = e^{y_{b,t}^\$}$. Because the only component of inflation that is priced is the default L_t, and because the price Q_t, yield y_b, and return $R_{b,t+1}$ assumed default, the only difference lies in expected inflation. That is, $Q_t^\$ = Q_t e^{-\mu_{\pi,t}}$ and $y_{b,t}^\$ = y_b + \mu_{\pi,t}$. Moreover, the expected return on the nominal bond, in real terms equals

$$\mathbb{E}_t\left[R_{b,t+1}^\$\frac{\Pi_t}{\Pi_{t+1}}\right] = \mathbb{E}_t[R_{b,t+1}].$$

The ex post inflation-adjusted rate in samples without disasters equals

$$\mathbb{E}[y_{b,t}^\$ - \Delta\pi_{t+1} \,|\, \text{no disasters}] = \mathbb{E}[y_b + \mu_{\pi,t} - \mu_{\pi,t}] = y_b. \tag{13}$$

This is the model counterpart of the average Treasury bill yield minus average realized inflation over the sample period of interest. The model with inflation differs from that of outright default in one respect: it allows for $\lambda < 0$, corresponding to the ability of nominal bonds to hedge inflation.

We now calibrate the model, keeping p constant at 2.25% (the calibrated value from 1984–2000), and allowing λ to vary. The form of Table 2 is the same as Table 1. We first show the price-dividend ratio and the inflation-adjusted Treasury yield for the two sample periods. Note that the model with inflation implies a different, and more precise, interpretation of the inflation adjusted-Treasury yield. The model counterpart is (13), whereas in the previous model it was simply the riskfree rate.

Analogously to the previous exercise, we fix all parameters other than consumption growth (which is taken from the data), patience β, and the inflationary default parameter λ. A higher λ corresponds to greater exposure, and hence a higher inflation premium. In line with the disaster literature, we consider a lower value of risk aversion γ (equal to 5) and a correspondingly larger disaster (a consumption decline of 30%). This calibration forms our benchmark; however, our points are qualitatively similar with higher γ and smaller disasters. We first consider the case of EIS equal to 2. We first note that the model is capable of matching the data, assuming a λ such that 13% of the bond value is lost in disasters in the first sample and essentially none in the second. Crucially, it does so with a a smaller increase in the discount rate β. Rather than 1.3 percentage points, β increases by 0.9 percentage points. This is because the model has a fundamentally different explanation for the decline in the interest rate, namely the reduced inflation premium. Indeed, the inflation premium (which we can calculate using (9)), is 1.4 percentage points in the first half of the sample, falling to negative 40 basis points in the second half, accounting for the majority of the decline in the observed interest rate.[10] An additional 35 basis points of the 3.5 percentage points reported

[10]Not surprisingly, this is a larger decline than that estimated using models assuming stationarity (see Favero et al. (2021) for a discussion of the role of the stationarity assumption in interest rate modeling). For instance, Haubrich et al. (2012) assume stationarity and Gaussian shocks, implying that the inflation risk premium depends directly on the variance of inflation. They find an average premium of 0.4%. Greenwald et al. (2022a,b) find similar results. Our model differs from these both in that we allow for a structural break and because we allow for rare inflation events, implying not only that true volatility is difficult to capture in-sample, but also that the premium does not depend solely on this volatility. That said, our estimation implies that, if averaged across the samples, the premium is 0.5%, which is not far from these prior estimates.

in Panel A arises from the fact that expected disasters did not occur (second term in (10)), leaving the remaining percentage point to come from a decrease in the real rate, driven by a combination of lower growth and higher patience.

Panel C assumes an EIS equal to 1. Note that, unlike the previous explanation, the model does not depend on an EIS greater than 1. Parameters in Panel C are similar to those in Panel B, with an even smaller rise in β. When the EIS is equal to one, the decline in growth now does not affect the price-dividend ratio (though it still affects the riskfree rate). Thus β now need increase by a mere 0.4 percentage points, because there is no need to counteract the effect of lower growth on the price-dividend ratio, and because growth also has a larger effect on the interest rate. To summarize, changes in λ help explain the decline in observed interest rates, rendering the assumption of a decline in disaster premia unnecessary. This explanation is more robust, in that it does not require a knife-edge combination of hard-to-observe risk, patience, and willingness to substitute across time.

3.4 Evidence for declining inflation risk

Is there any independent evidence that inflation risk has declined? A natural first place to look is at differences in yields on inflation-indexed and nominal bonds. The average difference in yields (break-even inflation) will combine both expected inflation and the inflation risk premium, among other factors (e.g., default through mechanisms other than inflation, like imperfect indexation). If we assume those other factors are reasonably small, then the difference between the inflation-adjusted nominal bond yield and the inflation-indexed bond yield is a direct proxy for the inflation risk premium.

Examining this difference in the U.S. would be ideal; however, given that Treasury Inflation-Protected Securities (TIPS) were first introduced in 1997 and did not have sufficiently liquid markets to measure riskfree returns until around 2004 (Fleming and Krishnan, 2004), these data are not available over our full sample. Index-linked Gilts from the UK,

Table 2: Accounting for the data with inflationary default risk

This table shows parameters necessary to match the data, assuming an endowment economy with rare disasters and inflationary default. We take average consumption growth from the data in each sample. We calibrate the discount factor β and the decline in bond value $\lambda\eta$ to match the average price-dividend ratio and the average inflation-adjusted Treasury bill, assuming no disasters. We vary the elasticity of intertemporal substitution (EIS) as shown. We assume the disaster probability equals 2.25%, its benchmark value in Table 1. Parameters and yields are in annual terms.

		Values	
	Parameter	1984–2000	2001–2021
Panel A: Moments in the data			
Price-dividend ratio	κ	42.34	50.86
Inflation-adjusted Treasury yield	y_b	0.0279	-0.0069
Panel B: $\gamma = 5$, EIS $= 2$, $\eta = 0.30$			
Average consumption growth	μ	0.0257	0.0148
Discount factor	β	0.973	0.982
Fraction of bond value lost	$\lambda\eta$	0.129	-0.036
Panel C: $\gamma = 5$, EIS $= 1$, $\eta = 0.30$			
Average consumption growth	μ	0.0257	0.0148
Discount factor	β	0.977	0.981
Fraction of bond value lost	$\lambda\eta$	0.129	-0.036

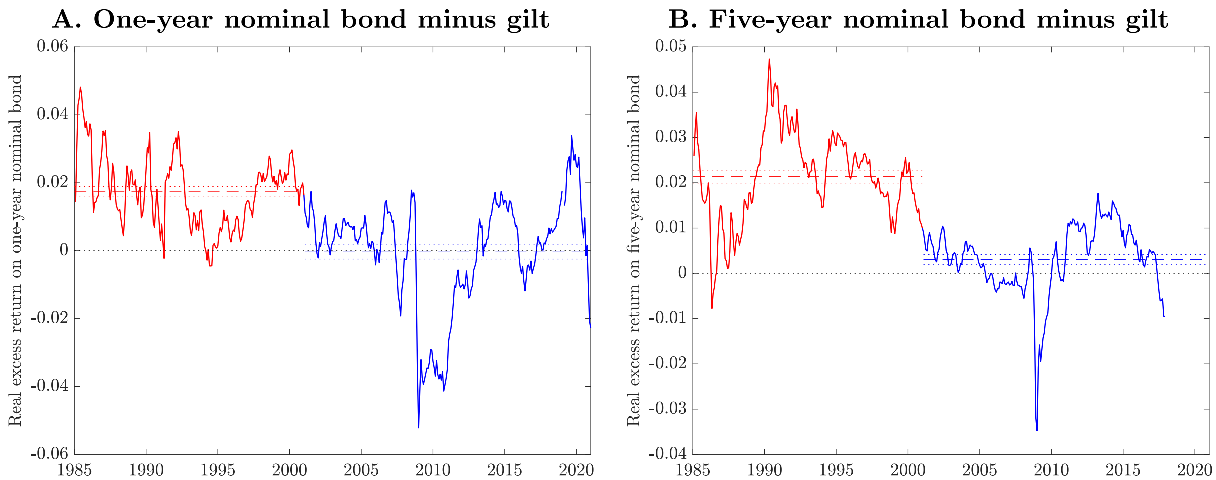
on the other hand, have traded since the early 1980s and provide the ideal asset to examine this difference over the last four decades.

Panel A of Figure 5 shows the difference between the inflation-adjusted yield on the one-year U.K. government nominal bond and the yield on five-year index-linked Gilts. The figure shows that, in the first sample half, this difference was significantly positive and large—almost 2 percentage points, nearly as large as the average Gilt yield itself. In the second sample half, in contrast, this excess return is on average zero, consistent with our estimate of no default risk. This suggests an economically large decline in the inflation risk premium.

A possible concern with this measure is that it compares bonds of different maturities.

Figure 5: Excess real returns on nominal bonds over inflation-indexed bonds

The figure shows the inflation-adjusted yields on nominal U.K. government bonds in excess of the five-year gilt yield. In Panel A, the solid line is the one-year inflation-adjusted nominal bond yield (the yield less realized inflation over the next year) minus the contemporaneous five-year gilt yield. Panel B is the same difference for the five-year nominal bond yield less the next five years of inflation (annualized). Dashed and dotted lines represent sample means with two-standard-error confidence intervals.



It may, for example, be that differences in these bond returns arise from changes in the term structure. Panel B addresses this concern by calculating the excess real return for five-year nominal bonds and subtracting (annualized) five-year realized inflation. The result is essentially identical: five-year nominal bonds earned a premium of over 2 percentage points in the 1980s and 1990s, and no premium in the 2000s and 2010s. Parameter values in Table 2 imply a strikingly similar decline of 2.5 percentage points, validating our model calibration.

Our analysis aims to understand long-run structural change *between* our sample periods, but it is worth mentioning that we also observe a significant reduction in both nominal and inflation-indexed yields *within* our second sample period. Panel A of Figure 6 displays the yield on five-year TIPS, while Panel B displays the yield on five-year index-linked Gilts. TIPS yields have decreased modestly since their introduction, while index-linked Gilts have fallen nearly 6 percentage points since 2001.[11] We also plot the equity valuation ratios in each respective country. Holding other factors constant, we would expect substantial declines in riskfree rates to be accompanied by increasing valuations. Puzzlingly, the observed yield declines did not coincide with rising valuations; indeed, in both the U.S. and the U.K., there is little correlation between the two series. Two potential offsetting differentials could be either an increased risk premium or declining expected growth. Neither of these explanations is supported by the data: the volatility indices for both the U.S. and U.K. have remained notably stable[12] and anticipated macroeconomic growth has only decreased marginally, by 1–1.5 percentage points.[13] Such a disconnect between valuation ratios and inflation-adjusted yields across various frequencies suggests the presence of other forces influencing asset prices.[14]

There are also reasons that standard break-even-based estimates of the sovereign default premium may be too low. The calculation above, for example, assumed that all default takes place through inflation. Non-zero prices on credit-default swaps suggests otherwise (Chernov et al., 2020).[15] Even if one were to relax this assumption (and allow an additional term for outright default), there remains two reasons why the estimate may be too low: (1) inflation indexation is inexact (or, more precisely, errors may be correlated with factors investors care about, such as inflation itself) and (2) recovery rates may be lower on inflation-indexed bonds. Indeed, TIPS explicitly do not index for deflation. Anderson and Sleath (1999) discuss errors in inflation indexation for Gilts. In response to a recent uptick of inflation, the government of Canada has discussed halting the issuance of inflation-linked securities due to

[11] A portion of the inflation-linked Gilt decline is likely attributable to the outsized demand for index-linked gilts from pension funds stemming from regulatory pressure in the late 1990s and early 2000s (National Association of Pension Funds, 2011). This means that the decline in index-linked Gilts could be artificially larger than the decline in the true riskfree rate.

[12] The stability of the VIX in the U.S. can be seen in Figure 4. The U.K.'s FTSE 100 VIX was 12.86 from 2004–2006 and 12.79 from 2017–2019 (the three most recent years of data we could locate).

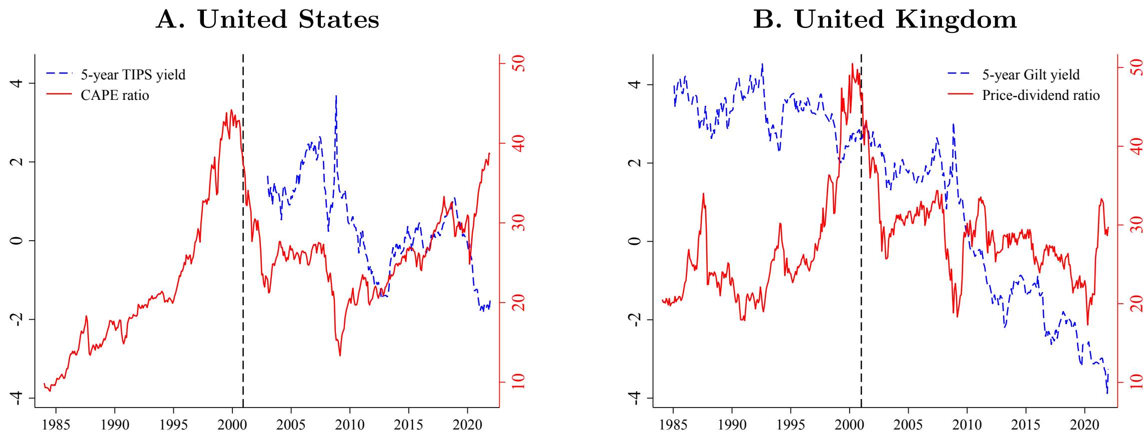
[13] The IMF, for example, forecasts real GDP growth in the U.K. at 1.5 percent, quite similar to the growth observed post-Financial Crisis. These data can be found here.

[14] For example, recent papers have studied the effects of segmented markets (Siriwardane et al., 2022) and rising convenience yields (Jiang et al., 2019, van Binsbergen et al., 2022)

[15] Some of the premium for default may reflect the probability of a temporary halt in payment (technical default), whereas the premium (9) assumes missed payments are not made at a later date (Bomfim, 2022).

Figure 6: Inflation-indexed bonds and equity valuation ratios

The figure shows yields on five-year inflation-indexed bonds and equity valuation ratios in the United States and United Kingdom. Information on the data and their sources can be found in Appendix A.

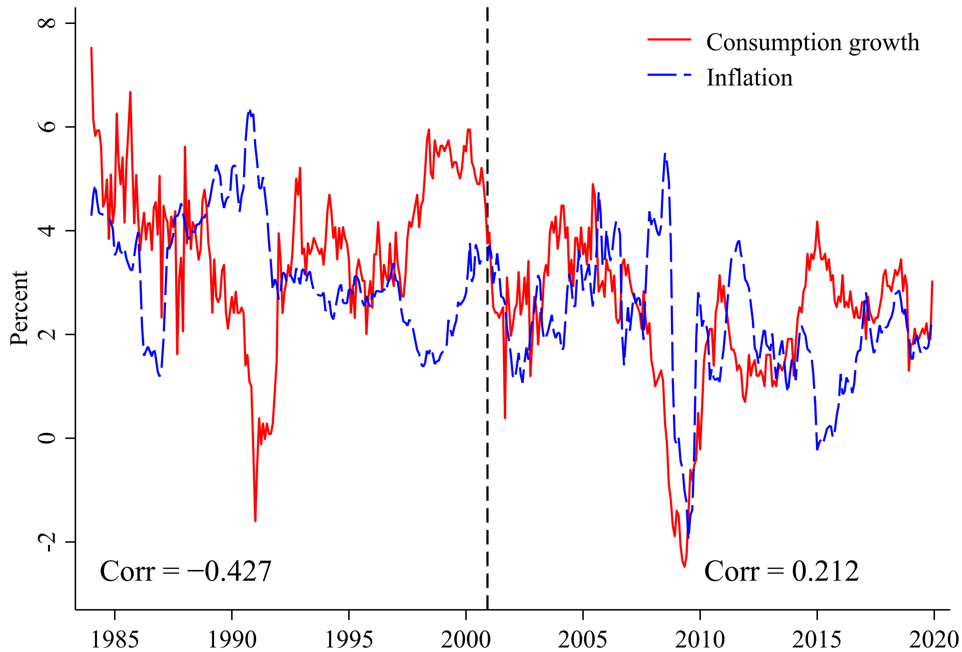


concerns regarding the size of the potential payouts (Czitron, 2022). Dittmar et al. (2019) provide evidence that yields price in greater default risk for TIPS than nominal Treasury bonds. Indeed, it is not known how investor expectations of indexing, and thus prices on inflation-indexed bonds, would behave in a setting with very high unexpected inflation.

Another place to look for evidence of a declining inflation risk premium is the sample correlations between inflation and consumption growth and between bond and stock returns. Ideally, to capture changes in λ, one would directly observe behavior during disasters, and use these observations to construct a correlation that is disaster-specific. That is the literal interpretation of the model in the previous section. However, both expected and realized behavior during disasters is hard to observe due to there being so few of them, and to a lack of a means to elicit expectations within them. Absent these ideal data moments, we look directly at realized inflation and consumption growth, with the view that the line between a downturn and disaster is ultimately arbitrary. Figure 7 shows the result. The recession in the early 1990s is clearly accompanied by inflation, whereas the boom of the late 1990s

Figure 7: Consumption growth and inflation in the United States

This figure shows one-year realized consumption growth and the one-year realized inflation rate from 1984 to 2019. The vertical line separates the sample periods at the beginning of 2001.



by deflation. However, the financial crisis was deflationary. A positive inflation-growth correlation is also consistent with the recent COVID recession, which saw deflation during the brief 2020 contraction followed by inflation and positive growth. In the first sample, the correlation is negative at -0.4; in the second sample it is positive at 0.2.

Also in the spirit of the model is the change in bond-stock correlations. Unexpected inflation that is negatively correlated with consumption will cause bond and stock returns to be positively correlated; unexpected inflation that is negatively correlated causes the reverse. Stock returns are driven in part by unexpected changes in dividends, which are correlated with consumption. Likewise, bond returns are negatively correlated with realized inflation. Indeed, Campbell et al. (2017) and Campbell et al. (2020) observe a substantial shift in the bond-stock beta, going from positive to negative over the two halves of the sample.[16] This relies on the same interpretation as the previous paragraph on changing correlations in downturns that may fall short of disasters. However, a changing bond-stock correlation also sheds light on what agents expect should disasters occur. An increase in the probability of disaster will cause both bond and stock prices to fall—assuming that inflation occurs in a downturn. Otherwise, bond prices will rise while stock prices fall. Thus, a change in the correlation is powerful evidence of how investors *expect* inflation and consumption to behave, which is what matters from the point of view of the model.

A final source of evidence for declining inflation risk comes from data on inflation expectations. Figure 8 shows a decline in inflation forecasts over four decades, leveling off in more recent years. Thus, as inflation fears receded, the perceived risk of inflation also receded—not only did inflation expectations decline, they also became less volatile. Indeed, Reis (2020) finds an anchoring of inflation expectations using survey data.

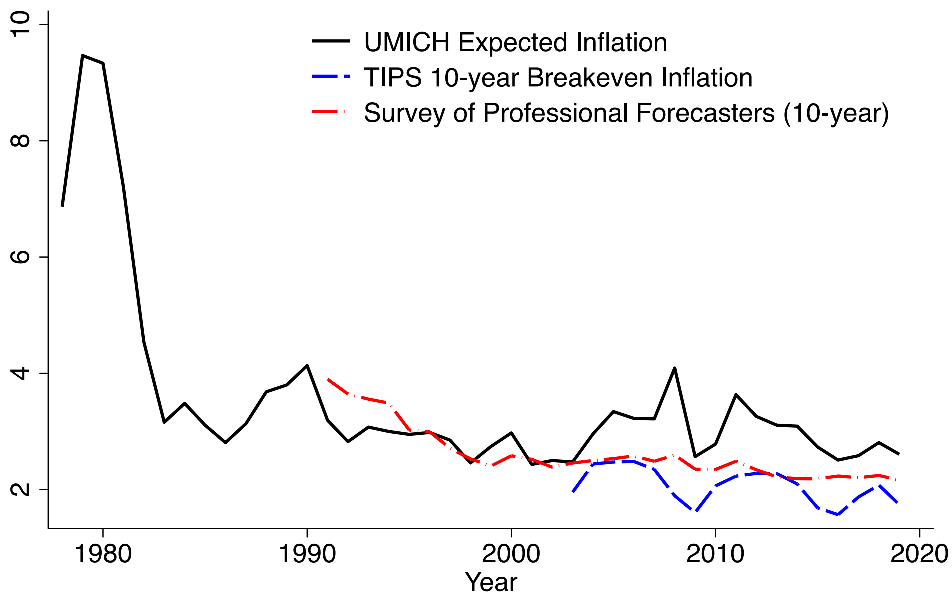
One can also infer inflation risk from expectation errors. Data on one-year-ahead forecasts from the Survey of Professional Forecasters also show that in the first sample, forecasters consistently over-estimate inflation, whereas in the second sample, their estimates are on average correct (Figure 9). Researchers have interpreted this difference as evidence of slow learning due to highly persistent underlying processes (Farmer et al., 2023) or the strong pull of past experience (Goetzmann et al., 2022), both of which is also in the spirit of our model. Regardless, in the first sample, investors forecasted inflation that did not occur, whereas in the second, they ceased to forecast inflation. This is consistent with a structural break in which, in the first sample, inflation exhibits positive skewness ($\lambda > 0$), that vanishes in the second ($\lambda \approx 0$).[17] Comparing estimates from Table 2 together with estimates from Figure 9

[16] Relatedly, Cieslak and Vissing-Jorgensen (2021) show that the so-called "Fed Put"—the tendency of the Fed to reduce rates (increasing bond prices) when the stock market falls—began only in the late 1990s.

[17] When agents predict disasters that do not occur, on average the difference between forecasted and the measured ex post average will be $\mathbb{E}_t[\Delta\pi_{t+1}] - \mu_{\pi,t} = -p\log(1-\lambda\eta) \approx p\lambda\eta$, which is positive in the first sample but zero in the second.

Figure 8: Expected inflation in the United States

The solid black line shows expected inflation from the Surveys of Consumers of University of Michigan. The dashed blue line shows the 10-year breakeven inflation rate computed from Treasury Inflation-Indexed Constant Maturity Securities. The dashed-dotted red line shows 10-year expected inflation from the Survey of Professional Forecasters.

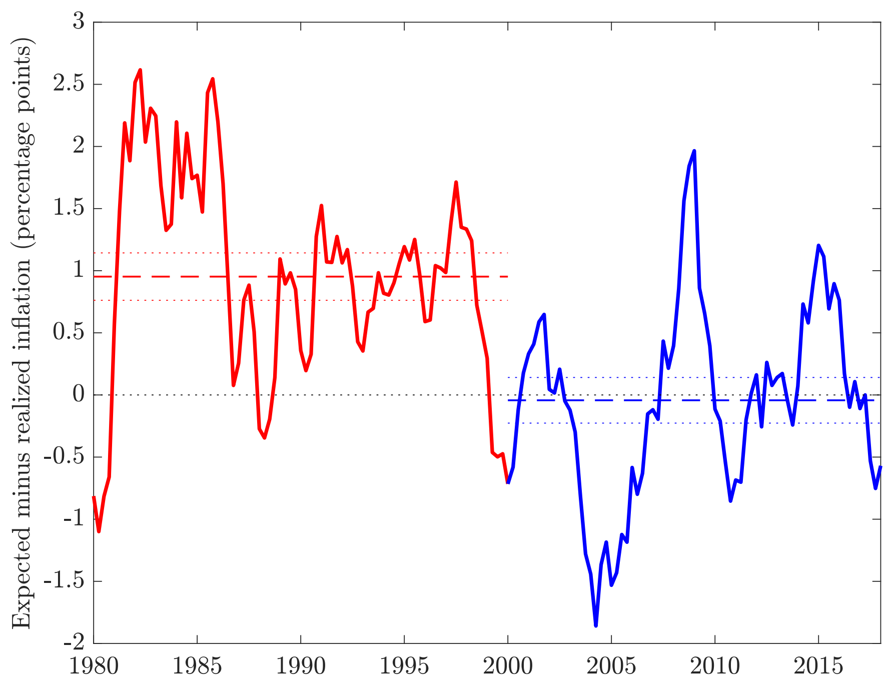


indicate that about a half of expectational "errors" are due the failure of disasters to occur.

To summarize, our model and calibration imply that the *true* riskfree rate and equity risk premium have remained relatively stable over time, a conclusion that is consistent with evidence from valuation ratios, which have remained relatively flat; and the VIX, which suggests no substantial increase in risk. The price-dividend ratio is unaffected by inflationary default risk. It is, however, common in the literature to use the return on the one-year government bond as a proxy for the true riskfree rate. Our calibration suggests that estimating the equity premium directly using this bond return implies an increase in the *measured* risk premium. In the model, this increase comes not from an increase in equity risk, but from a decline in the risk premium on government debt.

Figure 9: Expected versus realized one-year inflation

The figure plots the difference between expected and realized one-year inflation, where expectations are taken from the Survey of Professional Forecasters. The horizontal dashed lines show the average difference in each of our respective samples along with two-standard-error confidence intervals. These averages could be interpreted as estimates of $p\lambda\eta$ in our model, where p is the probability that a disaster occurs, and $\lambda\eta$ is the fraction of bond value lost when a disaster occurs.



This is *not* to say that riskfree rates have not declined at all. In the calibrations we present above, the riskfree rate declines from the first sample half to the second. This is mirrored in the data too: the yield on index-linked Gilts has also fallen. Our point is that this decline is substantially smaller than what is reflected in declines in inflation-adjusted nominal yields and accounting for this explains the joint evolution of valuation ratios and bond yields.

4 Production economy with inventory

Finally, we ask whether these results are robust to inclusion into a production economy, in which growth ultimately arises from firm productivity and depreciation. We introduce a novel element that naturally arises with low nominal interest rates: a riskless technology that allows goods to move from one period to another (inventory). The existence of this technology helps address another puzzle, that of the behavior of the investment-capital ratio.

We first solve a standard production economy model and show that it implies allocations and prices identical to an endowment economy. This allows us to more clearly show the effect of the inventory technology in the next section.

4.1 No-inventory case

We consider a standard production model in which capital quality can decline suddenly and unpredictably.[18] Let K_t denote the quantity of productive capital at time t. Given K_t and constant productivity A, output equals

$$Y_t = AK_t. \tag{14}$$

Let δ denote depreciation and X_t investment. Capital evolves according to:

$$\tilde{K}_{t+1} \equiv X_t + (1-\delta)K_t \tag{15}$$

$$K_{t+1} \equiv \tilde{K}_{t+1}(1-\chi_{t+1}), \tag{16}$$

where χ_{t+1}, defined in (2), represents destruction of capital. We assume $A > 1-\delta$, consistent with a growing economy. Following Gomes et al. (2019), we refer to $\tilde{K}$ as planned capital, the quantity of capital available if the disaster does not occur.

[18] See Barro (2009), Gabaix (2011), and Gourio (2012).

We can restate the agent's problem as a consumption-portfolio choice decision in which the agent allocates savings to capital and the riskfree bond. Let B_t denote the time-t dollar allocation to the riskfree asset. Define the agent's wealth at time t as

$$W_t \equiv C_t + B_t + \tilde{K}_{t+1}, \tag{17}$$

If investment in capital grows at the stochastic rate $R_{K,t+1}$, wealth at time $t+1$ must equal

$$W_{t+1} = B_t R_{f,t+1} + \tilde{K}_{t+1} R_{K,t+1}. \tag{18}$$

What is $R_{K,t+1}$? Equations (14–16) indicate that, should a disaster not occur, a single unit of capital creates A units of output. A fraction δ is lost prior to the next period. Should a disaster occur, then a fraction χ_{t+1} is lost. Given the remaining capital, A units of output are created and an additional fraction δ is lost. Therefore, the return on capital is

$$R_{K,t+1} = (1 - \delta + A)(1 - \chi_{t+1}). \tag{19}$$

We can rewrite the budget constraint in terms of flow variables. Equating (17) with (18) at time t and substituting in for $R_{K,t}$ implies

$$C_t + B_t + \tilde{K}_{t+1} = B_{t-1} R_{f,t} + \tilde{K}_t (1 - \delta + A)(1 - \chi_t).$$

Using (15) and (16), then subtracting $(1-\delta)K_t$ from both sides implies

$$C_t + B_t + X_t = Y_t + B_{t-1} R_{f,t}. \tag{20}$$

That is, output from the capital stock plus wealth in bonds can be used toward consumption, bond purchases at time t, or investment in the productive asset.

We can also rewrite the budget constraint in terms of the evolution of wealth. Define the share of savings invested in capital as

$$\alpha_t \equiv \frac{\tilde{K}_{t+1}}{W_t - C_t}.$$

Substituting in for B_t in (18) from (17) implies that

$$W_{t+1} = (W_t - C_t)(R_{f,t+1} + \alpha_t(R_{K,t+1} - R_{f,t+1})), \tag{21}$$

is an equivalent expression for the budget constraint. Let $R_{W,t+1} \equiv W_{t+1}/(W_t - C_t)$ denote the return on the wealth portfolio.

We assume Epstein and Zin (1989) and Weil (1990) preferences with unit EIS. The agent chooses consumption C_t and the capital portfolio share α_t to solve

$$\max_{C_t,\alpha_t} \left(C_t^{1-\beta} \left(\mathbb{E}_t \left[V(W_{t+1})^{1-\gamma} \right] \right)^{\frac{\beta}{1-\gamma}} \right), \tag{22}$$

subject to (21). Conjecturing that $V(W_t)$ equals a constant multiplied by W_t, and applying the first-order condition for optimal consumption implies the standard unit EIS result $C_t/W_t = 1 - \beta$.

In equilibrium, the bond is in zero net supply ($\alpha_t = 1$), and (20) reduces to

$$C_t + X_t = Y_t = AK_t. \tag{23}$$

Furthermore, the conditions $\alpha = 1$ and $C_t = (1 - \beta)W_t$ imply that consumption is a fixed percentage of planned capital:

$$C_t = \frac{1-\beta}{\beta}\tilde{K}_{t+1} = \frac{1-\beta}{\beta}(X_t + (1-\delta)K_t), \tag{24}$$

where the second equality follows from the capital accumulation equation (15).

What does this model imply for investment and for economic growth? Substituting in for C_t in (23) gives us the equilibrium investment-capital ratio with unit EIS:

$$\frac{X_t}{K_t} = \beta(1 - \delta + A) - (1 - \delta). \tag{25}$$

Evidently, the investment-capital ratio is strictly increasing in β. An increased demand for savings coming from an increase in β (a savings glut) unambiguously leads to an investment boom. Further, in this unit-EIS case, risk does not affect the investment decision: lower investment relative to capital must come through either a reduction in β or from the deterministic components of the return on capital A and δ. One may reconcile a decline in the riskfree rate with a decline in investment by arguing that productivity A or depreciation δ have declined. In order to match the decline in growth—a decline in μ in the endowment economy—one would need $A - \delta$ to decline as well. But even if this explanation succeeds at matching investment and interest rates, the puzzle of stable valuation ratios and the dependence of results on the EIS remain unresolved. If the EIS were to exceed 1, increased macroeconomic risk could lead to a reduction in X/K, but this relies on scant evidence of increased risk and requires placing economically meaningful restrictions on the EIS.

Consumption, investment, and output grow at the same rate. First, note that wealth grows at rate:

$$\frac{W_{t+1}}{W_t} = \frac{W_t - C_t}{W_t}\frac{W_{t+1}}{W_t - C_t} = \beta R_{K,t+1}. \tag{26}$$

(We have used the constant consumption-wealth ratio and the equilibrium condition $\alpha = 1$.) This must also be the growth rate of consumption. Substituting in for $R_{K,t+1}$ implies

$$\frac{C_{t+1}}{C_t} = \beta(1 - \delta + A)(1 - \chi_{t+1}). \tag{27}$$

This is then also the growth rate of planned capital, lagged one period. In equilibrium, all investment is in planned capital, so $\tilde{K}_{t+1}/\tilde{K}_t = W_t/W_{t-1}$. From (26), then, it follows that[19]

$$\frac{K_{t+1}}{K_t} = \frac{\tilde{K}_{t+1}}{\tilde{K}_t}\frac{1-\chi_{t+1}}{1-\chi_t} = \beta R_{K,t}\frac{1-\chi_{t+1}}{1-\chi_t} = \beta(1-\delta+A)(1-\chi_{t+1}).$$

The result for output then follows from $Y_t = AK_t$ and the result for investment follows from (23). As a consequence, the price-dividend ratio κ^Y on the claim to output equals the price-dividend ratio κ on the consumption claim: $\kappa^Y = \kappa = \beta/(1-\beta)$.

We now turn to the implications of this model for the interest rate and for stock returns. Given $V(W_t) \propto W_t$, the first-order condition with respect to α implies

$$\mathbb{E}_t\left[R_{W,t+1}^{1-\gamma}\right]^{-1}\mathbb{E}_t\left[R_{W,t+1}^{-\gamma}(R_{K,t+1}-R_{f,t+1})\right] = 0 \tag{28}$$

(see Appendix E for details). The equilibrium condition $\alpha = 1$ implies $R_{W,t+1} = R_{K,t+1}$. Substituting $R_{K,t+1}$ into (28) implies a value for the log riskfree rate:

$$\begin{aligned}\log R_f &= \log \mathbb{E}_t\left[R_{K,t+1}^{1-\gamma}\right] - \log \mathbb{E}_t\left[R_{K,t+1}^{-\gamma}\right] \\ &= \log(1-\delta+A) + \log(1+p((1-\eta)^{1-\gamma}-1)) - \log(1+p((1-\eta)^{-\gamma}-1)) \\ &\approx A-\delta+p((1-\eta)^{1-\gamma}-(1-\eta)^{-\gamma}).\end{aligned} \tag{29}$$

Equations (28) and (29) imply the following expression for the SDF:

$$M_{t+1} = \mathbb{E}_t\left[R_{W,t+1}^{1-\gamma}\right]^{-1} R_{W,t+1}^{-\gamma}. \tag{30}$$

[19]Here we have used the fact that planned capital $\tilde{K}_{t+1}$ is a constant fraction of wealth W_t (in this model, this fraction is one), so that $\tilde{K}_{t+1}/\tilde{K}_t = W_t/W_{t-1} = \beta R_{K,t}$.

Furthermore, the risk premium equals

$$\log \mathbb{E}_t[R_{K,t+1}] - \log R_f = \log(1 - p\eta) \\ + \log(1 + p((1-\eta)^{-\gamma} - 1)) - \log(1 + p((1-\eta)^{1-\gamma} - 1)), \quad (31)$$

exactly as in the endowment economy.

These asset pricing results are isomorphic to the endowment economy from Section 3. Indeed, equilibrium prices in the two models are identical if the parameters are such that the equilibrium consumption growth processes are the same.[20] The key difference between the models, however, is that there are two margins of adjustment in the production economy: quantities and prices. This is why, for example, the patience parameter β does not show up in (29). Instead, β influences quantities through the investment-capital ratio, which in turn affects prices. In the standard endowment economy, quantities cannot adjust, as the representative investor consumes whatever is produced in a given period.

4.2 Inventory case

Suppose now that, in addition to capital and a riskfree bond, the agent can put funds into inventory, namely a riskfree storage technology with a zero net return. If we impose the condition that riskfree storage be in zero supply, then the economy reduces to that in the previous section. The innovation in this section is that the inventory asset can be in positive supply across the economy.

Why would one have a positive-supply riskfree asset? As mentioned in the introduction, any store of value from one period to another could count as inventory, provided that it is in fact riskfree and can be frictionlessly interchanged between consumption and invest-

[20]In this setting, this occurs when $\beta^{-1}e^{\mu} = (1 - \delta + A)$. One can verify this by comparing (6) and (29). In general, production and endowment economies can be mapped to one another by equating the consumption processes, a fact which is discussed in Chapter 2 of Cochrane (2001).

ment. Many consumption goods would not fit this description because they cannot easily be changed into something other than what they are. Money does fit this description when the risk of unexpected inflation diminishes; therefore, we will think of inventory as money.

We acknowledge that categorizing inventory—modeled as a real asset—as money, a financial asset, requires a certain degree of conceptual abstraction. It is true that neither firms nor government agencies retain sufficient consumer goods in stock to fully substantiate the money supply. Nevertheless, there are solutions to reconcile this apparent incongruity. For example, it is not strictly necessary for firms or governments to maintain a tangible asset backup for their inventory, as long as they have the capacity to produce these goods in response to emerging demand. Ultimately, if people have faith that money can be securely held and readily exchanged for goods or services without significant inflation risk, the key implications of our model continue to hold.

Since we think of inventory as money, we must impose the condition that inventory can only exist as a feasible investment opportunity when inflation risk λ is low or negative. This means that our analysis of inventory applies only to the second sample period, in which we estimate low inflation risk. This turns out to make no difference—in the first sample, when the equilibrium interest rate is greater than zero, inventory can exist but agents choose not to hold it.[21] Again, strictly speaking, if the inventory asset is cash and there is expected inflation but no unexpected inflation, then we could specify a negative expected return on the inventory asset. However, expected inflation in the second sample period is small, so allowing for a slightly different return on inventory would make little difference.

Like all valuation equations, the existence of this riskfree storage is predicated on investors' (subjective) expectations about inflation. Evidence suggests (Reis, 2020) that investors believed inflation would be low and stable, and thus consistent with our assumptions

[21] Liquidity services could be one reason investors choose to hold inventory in the presence of a positive riskfree interest rate; this is the case in inventory-theoretic models like that of Alvarez et al. (2009). For simplicity, we do not assume these.

on the existence of inventory. As discussed above, interpreting inventory as money naturally relates this safety technology to the safety of the government, as in Blanchard (2019). Consequently, the fiscal theory of the price level (Cochrane, 2021) could provide a foundation for investors' beliefs in low and stable inflation.[22]

Consider the agent's problem in Section 4.1, except here the agent can invest in a storage technology with quantity I_t. The agent maximizes unit-EIS recursive utility by choosing consumption and B_t, I_t, and $\tilde{K}_{t+1}$. That is, the agent recursively solves

$$\max_{C_t, B_t, I_t, \tilde{K}_{t+1}} \left(C_t^{1-\beta} \left(\mathbb{E}_t \left[V(W_{t+1})^{1-\gamma} \right] \right)^{\frac{\beta}{1-\gamma}} \right), \tag{32}$$

subject to

$$W_t = C_t + B_t + I_t + \tilde{K}_{t+1} \tag{33}$$

$$W_{t+1} = B_t R_{f,t+1} + I_t + \tilde{K}_{t+1} R_{K,t+1} \tag{34}$$

$$I_t \geq 0. \tag{35}$$

The solution is still characterized by $V(W_t) \propto W_t$, implying the result $C_t/W_t = 1 - \beta$.

We now characterize the equilibrium. We will use the notation R_f^* to denote the equilibrium interest rate in the no-inventory economy (Section 4.1, Equation 29). Then:

1. If $R_f^* > 1$, then in equilibrium $I_t = 0$, and the equilibrium is the same as in Section 4.1.

2. If $R_f^* < 1$, then $I_t > 0$. Investment in inventory crowds out investment in productive capital.

[22] While our paper says nothing explicit about the accumulation of government debt, an interesting question for future work is how low rates and inventory affect optimal government policy. A declining premium on government debt could be one major reason that debt-to-GDP increased: as perceived safety increases and the discount rate declines, the government could, ostensibly, sustain a higher level of debt. Inventory demand provides another reason for higher debt. Equivalently, these channels can explain why rising debt did not result in higher default risk premia. Blanchard (2019) provides a related argument to rule out default risk as a concern for investors.

The argument is as follows (Appendix E gives an alternative, more formal proof). First, consider the case of $R_f^* > 1$, and conjecture that $R_{f,t+1} = R_f^*$ constitutes an equilibrium in (32–35). This investor would never choose $I_t > 0$ because bonds offer superior returns; on the other hand (35) implies that the agent cannot short-sell inventory. Therefore $I_t = 0$, namely, the inventory asset is irrelevant, and thus $\alpha = 1$ is still the market-clearing condition. Equilibrium quantities and returns are the same as in Section 4.1.

Now assume that $R_f^* < 1$. The only possible equilibrium value for R_f is unity. This is because $R_f < 1$ implies an arbitrage opportunity: the investor would borrow at R_f and invest the proceeds in the inventory asset. If instead $R_f > 1$, the reasoning in the above paragraph implies the agent would hold no inventory. That means $R_f^* = R_f > 1$, contradicting the assumption. Intuitively, we can find an equilibrium with inventory for the following reason: if the agent does not hold inventory ($\alpha = 1$) and the riskfree rate equals $R_{f,t+1}^* < 1$, then the agent will wish to hold more inventory, as it is a marginally better asset. Doing so, however, reduces the volatility of the return on the wealth portfolio and stochastic discount factor and thus increases the equilibrium riskfree rate. The agent will increase holdings of inventory until the equilibrium rate is equal to the return on inventory. The power of this reasoning is that we can proceed by analyzing the cases $R_f^* < 1$ and $R_f^* > 1$ separately. Equation (29) indicates that low productivity A, high depreciation δ, risk-averse investors, and high risk of disasters might lead to R_f^* falling below one.

We focus on the case of $R_f^* < 1$; as the above argument shows, this is where inventory matters. We show it is also empirically relevant in that it prevails in the second sample period. Bonds are redundant, so we can assume $B_t = 0$. The requirement $R_f = 1$ replaces $\alpha = 1$ as the market-clearing condition. Given that the equilibrium takes this form, for convenience we can rewrite the agent's optimization problem as

$$\max_{C_t, \alpha_t} \left(C_t^{1-\beta} \left(\mathbb{E}_t \left[V(W_{t+1})^{1-\gamma} \right] \right)^{\frac{\beta}{1-\gamma}} \right),$$

subject to

$$W_{t+1} = (W_t - C_t)(1 + \alpha_t r_{K,t+1}),$$

where $r_{K,t+1} = R_{K,t+1} - 1$ is the net return on capital and α is the share of capital in savings, as it was in Section 4.1. The first-order condition for α continues to be (28), which, in the case with inventory, becomes

$$\mathbb{E}_t \left[\frac{1}{(1 + \alpha r_{K,t+1})^\gamma} r_{K,t+1} \right] = 0. \tag{36}$$

Thus far we have not imposed distributional assumptions. Given our assumption on χ_t, we obtain:

$$\frac{p r_{K,\eta}}{(1 + \alpha r_{K,\eta})^\gamma} + \frac{(1-p) r_{K,0}}{(1 + \alpha r_{K,0})^\gamma} = 0, \tag{37}$$

where (with some abuse of notation) we let $r_{K,0} \equiv (1-\delta+A)-1$ and $r_{K,\eta} \equiv (1-\delta+A)(1-\eta)-1$ denote the net returns on capital in the non-disaster and disaster states, respectively. Solving for α implies:

$$\alpha = \min \left\{ 1, -\frac{((1-p)r_{K,0})^{1/\gamma} - (-p r_{K,\eta})^{1/\gamma}}{((1-p)r_{K,0})^{1/\gamma} r_{K,\eta} - (-p r_{K,\eta})^{1/\gamma} r_{K,0}} \right\}, \tag{38}$$

The investor holds inventory when expected risk-adjusted capital returns are sufficiently low.

Because the consumption-wealth ratio is again $1 - \beta$, we can apply the same reasoning used to show (27) to find:

$$\frac{C_{t+1}}{C_t} = \frac{W_{t+1}}{W_t} = \beta(1 + \alpha r_{K,t+1}) = \beta \left(\alpha(1 - \delta + A)(1 - \chi_{t+1}) + 1 - \alpha \right). \tag{39}$$

Relative to the model in Section 4.1, consumption growth is less volatile because, in aggregate, agents use inventory to smooth aggregate fluctuations. It is also, on average, lower, because less is invested in the productive asset. Output growth, however, is *more* volatile. Consumption growth is no longer tethered to output as in Section 4.1. Still, the relation between growth in the capital stock and growth in wealth remains the same:

$$\frac{K_{t+1}}{K_t} = \frac{\tilde{K}_{t+1}}{\tilde{K}_t}\frac{1-\chi_{t+1}}{1-\chi_t} = \frac{W_t}{W_{t-1}}\frac{1-\chi_{t+1}}{1-\chi_t}.$$

Substituting in from (39) then implies

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \beta\left(\alpha(1-\delta+A)(1-\chi_{t+1}) + (1-\alpha)\left(\frac{1-\chi_{t+1}}{1-\chi_t}\right)\right), \tag{40}$$

Output growth is more volatile than consumption growth because it bears the full brunt of disasters: note that $1-\chi_{t+1}$ multiplies both the term with α (representing investment in the risky technology) and $1-\alpha$. By definition, the disaster applies to the entire existing capital stock. While this effect makes output growth more volatile than consumption in the present model, it does not, by itself, raise the volatility relative to the model in Section 4.1. There is, however, a second effect, represented by $1-\chi_t$ in the denominator. Coming out of a severe recession featuring capital destruction $\chi_t > 0$, output growth is higher because agents invest more to get back to the optimal allocation. This raises the volatility of output growth relative to the model in Section 4.1.

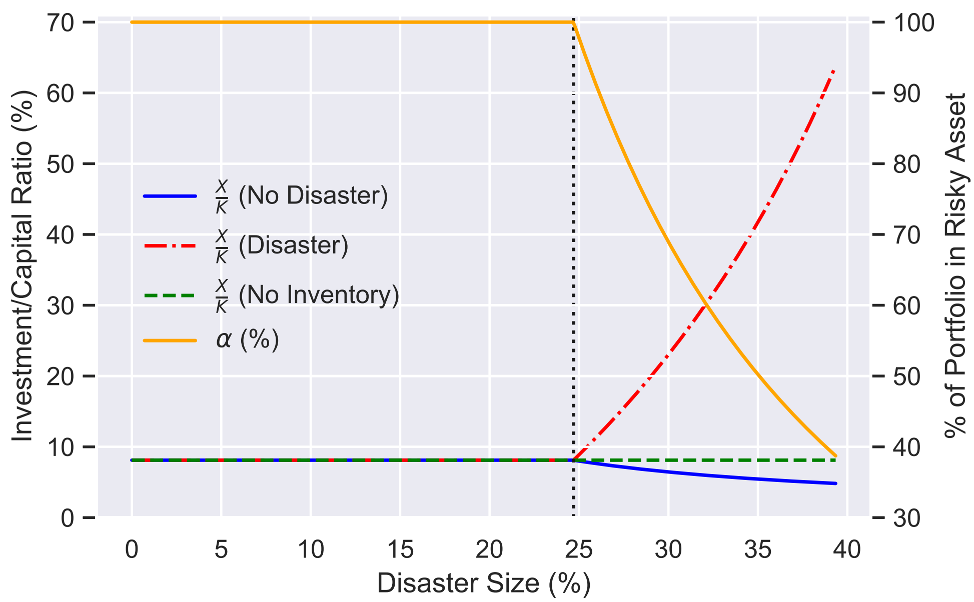
What are the properties of investment? Rewriting the capital accumulation equation (15) so that X_t is on the left-hand side, and dividing by K_t implies

$$\begin{aligned}
\frac{X_t}{K_t} &= \frac{\tilde{K}_{t+1}}{\tilde{K}_t}\frac{\tilde{K}_t}{K_t} - (1-\delta) \\
&= \beta R_{W,t}(1-\chi_t)^{-1} - (1-\delta) \\
&= \beta(\alpha(1-\delta+A) + (1-\alpha)(1-\chi_t)^{-1}) - (1-\delta),
\end{aligned}$$

where we have used the fact that $\tilde{K}_{t+1}/\tilde{K}_t = \beta R_{W,t}$. After capital disasters, the agent invests at a higher rate to replenish the capital stock. Consequently, the investment-capital ratio is time-varying in this economy, despite i.i.d. shocks and a balanced growth path.

Figure 10: Investment capital ratio in the model

The figure shows how capital investment varies with the size of the consumption decline in a disaster for the production models with and without inventory. The figure plots the investment-capital ratio X/K in the model with inventory when there is and is not a disaster, and in the model without inventory. It also plots α, the share of savings invested in capital. Risk aversion $\gamma = 6$, the EIS $\psi = 1$, the patience parameter $\beta = 0.963$, depreciation $\delta = 0.064$, the probability of disaster $p = 0.03$, and the marginal product of capital $A = 0.12$. The dotted black line represents the point at which the riskfree rate is equal to 0 in the model without inventory.



A disaster in the prior period increases investment in productive capital. This is because the disaster affects capital disproportionately, and the agent must re-invest to return capital back to its pre-crisis level. For an illustration, see Figure 10, which shows the investment-capital ratio for $\chi_t = 0$ (no disaster) and $\chi_t = \eta$ (disaster) for various values of the disaster size.[23] The figure also shows the optimal planned capital to wealth ratio α. For comparison, the figure also shows quantities in the case of no inventory. Fixing other parameters, for disaster sizes of less than 25%, the gross riskfree rate is above one, implying that the

[23]A higher disaster size has the same effect in the model as a higher disaster probability. What matters for these mechanism is total disaster risk.

economies with and without inventory are the same. As the size of the disaster increases, the equilibrium riskfree rate in the no-inventory economy falls sharply (we illustrate this in Figure 12). It becomes optimal to hold inventory and investment in productive assets falls. At that point, investment depends on the occurence of a disaster in the prior period. The greater the size of the disaster, the greater the increase in investment. In contrast, with no inventory, the investment-capital ratio is always the same.

We define the stock market as the claim to output Y_t in all future periods. As (40) shows, the growth rate of capital is no longer i.i.d. but depends on χ_t (note that χ_{t+1} is i.i.d. given time-t information). Therefore, the price-dividend ratio on the output claim is a function of χ_t and solves

$$\kappa^Y(\chi_t) = \mathbb{E}_t \left[M_{t+1} \left(1 + \kappa^Y(\chi_{t+1})\right) \frac{Y_{t+1}}{Y_t} \right],$$

where the stochastic discount factor takes the same form as (30), with $R_{W,t+1}$ now given as above. Note that $R_{W,t+1}$ is i.i.d. Under our distributional assumptions:

$$\kappa^Y(0) = \frac{\beta}{1-\beta} \left(\nu + (1-\nu) \left(\frac{1+\alpha r_{K,0}}{1+\alpha r_{K,\eta}} \right) (1-\eta) \right), \tag{41}$$

$$\kappa^Y(\eta) = \frac{\beta}{1-\beta} \left((1-\nu) + \nu \left(\frac{1+\alpha r_{K,\eta}}{1+\alpha r_{K,0}} \right) (1-\eta)^{-1} \right), \tag{42}$$

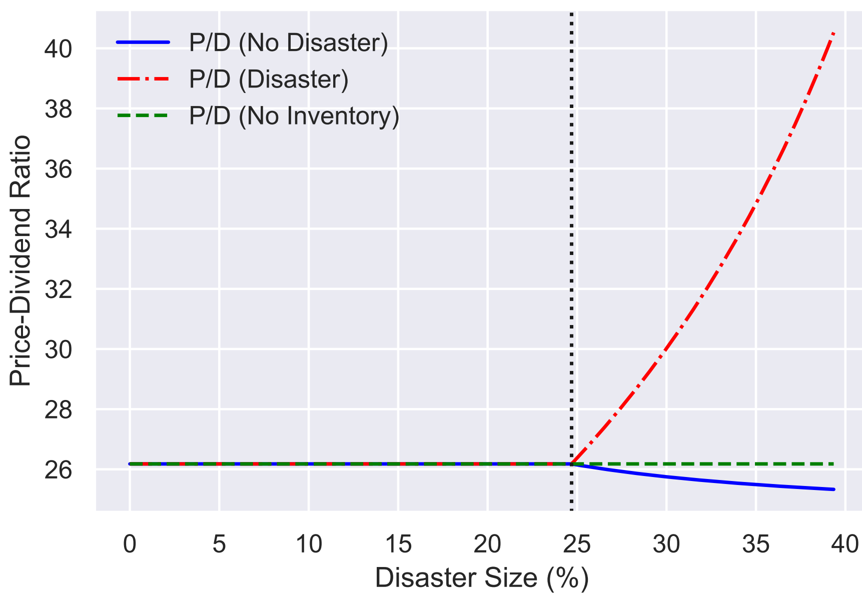
where $\nu \equiv ((1-p)(1+\alpha r_{K,0})^{1-\gamma})/((1-p)(1+\alpha r_{K,0})^{1-\gamma} + p(1+\alpha r_{K,\eta})^{1-\gamma})$. See Appendix E for details. In the case where $\alpha = 1$, the price-dividend ratio is the constant $\kappa^Y = \beta/(1-\beta)$.

Figure 11 shows the price-dividend ratio for various levels of the disaster size, both in the economy with inventory and in the economy without. The economy without inventory has a constant price-dividend ratio solely determined by β. When there is inventory, the price-dividend ratio rises in disasters because dividends are temporarily depressed (they are also low because of the disaster). This increase is due to the endogenous investment response, whereby inventory is liquidated after a disaster to rebuild the capital that is destroyed.

In contrast with standard production models, the price-dividend ratio in the no-disaster

Figure 11: Price-dividend ratio in the model

The figure shows how the price-dividend ratio varies with the size of the consumption decline in a disaster for the production models with and without inventory. The figure plots the price-dividend ratio in the model with inventory when there is and is not a disaster, and in the model without inventory. Risk aversion $\gamma = 6$, the EIS $\psi = 1$, the patience parameter $\beta = 0.963$, depreciation $\delta = 0.064$, the probability of disaster $p = 0.03$, and the marginal product of capital $A = 0.12$. The dotted black line represents the point at which the riskfree rate is equal to 0 in the model without inventory.



case declines (in a comparative statics sense) as a function of the disaster size (see Figure 11). In the case without inventory, the price-dividend ratio is independent of disaster risk. Models with production that seek to match business-cycle fluctuations in investment and valuation ratios require the EIS to be greater than 1. Endowment models achieve the same effect by imposing exogenous leverage (dividends more sensitive to shocks than consumption). In this model, leverage is endogenous, and qualitatively correct price-dividend ratio dynamics could in principle occur, even with an EIS of one. The magnitude of the decline in Figure 11 suggests that the effect is small under our calibration.

Figure 12 shows that the equity risk premium in this economy loses its usual dependence on disaster risk. The equity premium equals $rp \equiv \log \mathbb{E}_t[R_{Y,t+1}] - \log R_f$, where the return on the output claim is

$$R_{Y,t+1} = \left(\frac{\kappa^Y(\chi_{t+1}) + 1}{\kappa^Y(\chi_t)}\right)\left(\frac{Y_{t+1}}{Y_t}\right).$$

The blue line in the figure shows the equity premium in the model without inventory: it is highly dependent on the disaster size, as is the riskfree rate. However, the return on capital—which, in the economy with no inventory, is the equity return—is only very slightly decreasing. This is a standard result in disaster-risk economies: the full discount rate on the equity claim decreases slightly with the probability of a disaster.

While this might seem counterintuitive, it arises from the fact that, while the equity premium increases, the riskfree rate declines and more than offsets the effect. Also recall that the continuously compounded return in a standard i.i.d. economy can be expressed as the log dividend yield plus the log growth in cash flows. When the EIS equals one, the dividend yield does not depend on disaster risk, and so the only effect is the small effect of expected cash flows. In the economy with inventory, the return on capital is the same as in the economy without (this is defined by the production opportunities), and thus is slightly decreasing. The riskfree rate is constant, implying that the premium on capital is also slightly decreasing. The equity premium decreases slightly more in the disaster size as compared to $\log \mathbb{E}[R_K] - \log R_f$. This is because the increase in the price-dividend ratio counteracts the decline in output due to the disaster.

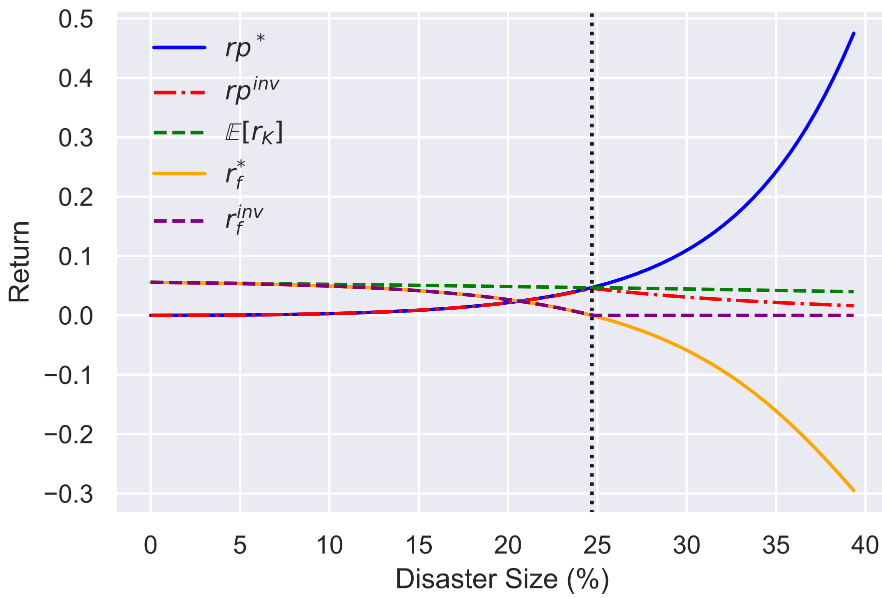
Finally, the inflation-adjusted Treasury yield is equal to

$$\begin{aligned} y_b = \log &\left(p(1+\alpha r_{K,\eta})^{1-\gamma} + (1-p)(1+\alpha r_{K,0})^{1-\gamma}\right) \\ &- \log\left(p(1+\alpha r_{K,\eta})^{-\gamma}(1-\lambda\eta) + (1-p)(1+\alpha r_{K,0})^{-\gamma}\right). \end{aligned} \tag{43}$$

While the true riskfree rate cannot go below zero, the yield and expected return on the defaultable claim could be positive or negative, depending on the sign of the risk premium.

Figure 12: Risk premia and riskfree rate in the model

The figure shows how the riskfree rate and risk premium vary with the size of the consumption decline in a disaster for the production models with and without inventory. The moments plotted are: the equity premium in the models with and without inventory, rp^{inv} and rp^{*}; the riskfree rates in the models with and without inventory, r_f^{inv} and r_f^{*}; and the expected return on capital, $\mathbb{E}[r_K]$. The equity premium is defined as the log expected return on the output claim minus the log riskfree rate. Risk aversion $\gamma = 6$, the EIS $\psi = 1$, the patience parameter $\beta = 0.963$, depreciation $\delta = 0.064$, the probability of disaster $p = 0.03$, and the marginal product of capital $A = 0.12$. The dotted black line represents the point at which the net riskfree rate is equal to 0 in the model without inventory.



The model is calibrated to match the inflation-adjusted Treasury yield, price-dividend ratio, and GDP growth in the U.S., as in the sections above. Calibrating to match these data requires solving a system of three equations in three unknowns, where the unknowns are the parameters β, λ, and A and the three equation are Equations (41), (43), and

$$\frac{Y_{t+1}(0)}{Y_t(0)} = \beta\left(\alpha(1 - \delta + A) + (1 - \alpha)\right) \tag{44}$$

which is GDP growth when the disaster does not occur. Indeed, each of the moments to

Table 3: Inventory and inflationary default in a model with production

The model is solved with risk aversion $\gamma = 5$ and EIS $\psi = 1$. Consumption declines 30% in a disaster ($\eta = 0.30$), the probability of disaster $p = 2.25\%$, and depreciation $\delta = 0.05$.

	Parameter	Values 1984–2000	Values 2001–2021
Panel A: Moments in the data			
Price-dividend ratio	κ	42.34	50.86
Inflation-adjusted Treasury yield	y_b	0.0279	-0.0069
US GDP growth	$\frac{\Delta Y}{Y}$	0.0241	0.0122
Panel B: Calibration, $\gamma = 5$, EIS $= 1$, $\eta = 0.30$			
Discount factor	β	0.977	0.981
Fraction of bond value lost	$\lambda\eta$	0.142	-0.06
Capital productivity	A	0.099	0.084

which we calibrate parameters is the value in the no-disaster state ($\chi_t = 0$), consistent with the fact that we do not observe any disasters in our sample. We then solve for the values of the parameters that equate the data moments with their corresponding model moments.

Table 3 displays the results from the calibration with inventory. The model explains the data moments with a quantitatively reasonable calibration of β, λ, and A. The slight increase in β matches the modest rise in the price-dividend ratio; lower capital productivity A matches the lower growth in the second sample.[24] Inflationary default risk $\lambda\eta$ falls, in line with the estimates in Section 3.[25] Notably, because the model does not require a substantial increase in β to explain asset prices—a force that would drive up investment and thus economic growth—the model estimates a much smaller decrease in productivity than would be required in the presence of a standard savings-glut mechanism.

As we know from our endowment-economy results, the model with sovereign default can

[24] We could, equivalently, keep A constant and estimate an increase in δ; they are isomorphic for explaining the growth decline.

[25] The estimates of λ are slightly different than in Table 2 because we calibrate to average GDP growth instead of consumption growth.

Table 4: Inventory and inflationary default with production: untargeted moments

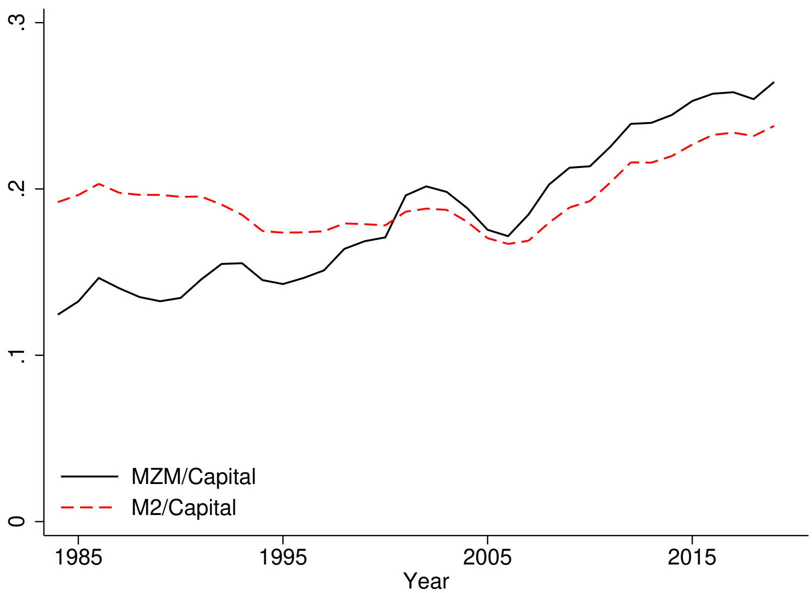
The model is solved with risk aversion $\gamma = 6$ and EIS $\psi = 1$. Consumption declines 30% in a disaster ($\eta = 0.30$), the probability of disaster $p = 2.25\%$, and the marginal product of capital $A = 0.12$. The calibrated parameters from Table 3 are used for β, λ, and δ.

		Values	
	Parameter	1984–2000	2001–2021
Panel A: With inventory, $\gamma = 5$, EIS = 1, $\eta = 0.30$			
Risky capital share	α	1.000	0.947
US GDP growth	$\frac{\Delta Y}{Y}$	0.024	0.012
Investment-capital ratio	$\frac{X}{K}$	0.074	0.062
Unconstrained riskfree rate	r_f^*	0.011	-0.003
Panel B: Without inventory, $\gamma = 5$, EIS = 1, $\eta = 0.30$			
Risky capital share	α	1.000	1.000
US GDP growth	$\frac{\Delta Y}{Y}$	0.024	0.014
Investment-capital ratio	$\frac{X}{K}$	0.074	0.064
Unconstrained riskfree rate	r_f^*	0.011	-0.003

explain these data moments with or without inventory. What is novel to the inclusion of inventory in this production model is its endogenous effects on investment and growth. Panel A of Table 4 reports model-implied moments for the inventory model under the calibration in Table 3. Due to both a decline in A and an endogenous decline in the capital share α, and despite a modest rise in β, the investment-capital ratio falls from 7.4% to 6.2%. Output growth falls with investment. To illustrate the role of inventory, Panel B of Table 4 shows what would have happened to growth and investment in the absence of inventory—that is, under the calibration in Table 3 but imposing $\alpha = 1$. Without the endogenous saving response at the zero lower bound, the effect of the decline in A on growth and investment are diminished. Notably, the model without inventory still predicts a decline in investment and growth because the increase in β is not large enough to induce an investment boom.

Figure 13: Money supply relative to the capital stock

The figure shows the ratio of the money supply to the capital stock in the U.S. economy. We report two measures of the money supply. The first is M2, which adds to M1 savings accounts, small time deposits, and retail money market mutual funds. The second is MZM (zero-maturity money), which is constructed by the Federal Reserve Bank of St. Louis and includes M2 less small-denomination time deposits plus institutional money market funds.



According to the model, small structural changes in the economy—including technological forces that depressed growth and demand-side forces that increased savings—drove down the unconstrained riskfree rate below zero. The existence of inventory amplified these structural changes: when the riskfree rate hit zero, investors began to hoard money, further driving down investment and growth and preventing interest rates from becoming negative. In summary, to account for declining interest rates, stable valuations, and stagnating investment and growth, we need a substantial decline in the inflation risk premium. Additionally, real interest rates fell to zero, precipitating a deepening of the economy's secular stagnation.

Table 4 suggests that, according to the inventory model, the unconstrained riskfree rate fell to approximately 30 basis points below zero, incentivizing investors to hold about 5% of their wealth in inventory. If inventory is indeed money, then this should show up in the data as an increase in the money supply relative to the capital stock. To assess this prediction of the inventory model, we plot this ratio in Figure 13. We report two definitions of the money supply, both of which have risen relative to the capital stock in the twenty-first century. The first is the common M2 measure. The second, which we argue is a better measure of inventory, is the more inclusive "zero-maturity money" measure (MZM) from the St. Louis Fed, which takes M2, removes small illiquid time deposits, and adds institutional money market mutual funds (whereas M2 only includes retail money market funds). The rise in MZM relative to capital over the past two decades has been sizable, in line with our model's prediction of an increasing share of wealth in money-like inventory assets.

5 Concluding remarks

The puzzle of declining interest rates is a puzzle not only from the point of view of the last quarter century, but over a much longer horizon. It is also a joint puzzle: why have low interest rates not been accompanied by higher valuation ratios and investment rates?

The purpose of this article is to argue that the most natural explanation is not an increased demand for savings, which would lower interest rates and raise valuation ratios; nor a decrease in growth, which is hardly enough on its own to account for the observed change; nor an increase in the risk premium, as there is no evidence that risk has increased by nearly the required amount. These joint phenomena have a simple explanation: the true riskfree rate has not fallen nearly as much as conventional measurements from nominal yields suggest. Government debt claims are defaultable, and investors have come to require a lower premium for this risk of default.

In support of our explanation, we build a framework to explain very low nominal debt yields that is also consistent with an equilibrium zero lower bound. We accomplish the former using a model with a risk of rare disasters. In a rare disaster model, investors' precautionary savings demand pushes the riskfree rate below zero. We accomplish the latter by introducing a costless storage technology into a production economy. When parameters are such that the true riskfree rate is below zero, agents choose to save into inventory until markets clear at a riskfree rate of zero.

What we do not model is the cause for the decline in investor expectations of sovereign default. Evidence suggests that this decline has both a relatively short-term component based on the history of the last 40 years and a long-term component spanning centuries, based on a growing faith over time in the stability of sovereigns. The forces determining this shift in expectations, at both high and low frequencies, are an interesting topic for further research.

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A Data description

We use various series to illustrate the secular decline in interest rates in the short- and long-run. To obtain interest rates from 1311–2018, we rely on data from Schmelzing (2020). The dataset contains nominal interest rate and inflation time series for several developed economies over the last eight centuries. Specifically, the data include long-term sovereign borrowing rates with an average maturity that hovers around 10 years; however, this varies over time and across countries. From these data, we plot the nominal sovereign borrowing yields for the United Kingdom, Holland, Germany, Italy, and the United States in Panel A of Figure 1. The data are collected from a variety of sources, outlined in detail in the paper and online appendix. The U.K. borrowing rates come from the Calendar of State Papers and the Bank of England. Data before 1694 for the U.K. (before the founding of the Bank of England) are not used, since the data are incomplete. Data for the Netherlands come from Dormans (1991), Weeveringh (1852), the European Central Bank, and various sources from Leiden, Haarlem, Utrecht, Schiedam, and Amsterdam. German data come from various sources from several German principalities. U.S. data come from Durand and Winn (1947), Homer and Sylla (2005), the NBER Macrohistory database, and Federal Reserve Economic Data (FRED) from the Federal Reserve Bank of St. Louis.

We also report the Bank of England (BoE) short-term lending rate (series BOERUKM) from FRED. From 1694 to 1971, the "bank rate" is used; from 1972 to 1981, the minimum lending rate is used; from 1981 to 1997, the BoE base rate is used; and from 1997 to the present, the BoE Operational interest rate is used. For more information see the Bank of England research datasets webpage.

Data for U.S. interest rates from 1984 to 2021 come from FRED. Our main measure for nominal interest rates in the U.S. is the effective Federal Funds Rate (series FEDFUNDS), the rate corresponding to the median volume of overnight unsecured loans between depository institutions. This is plotted in Panel A of Figure 2. In our calibration exercises, for comparability with Farhi and Gourio (2018) we use the one-year constant maturity Treasury rate, less current inflation. Data for U.K. interest rates from 1984 to 2016 come from Jordà et al. (2019), who in turn use data from Zimmermann (2017) and the Bank of England, who use the average rate on 3-month Treasury bills.

U.K. inflation-linked and nominal Gilts yields are taken from Global Financial Data, which sources the yields from the Bank of England. Inflation in the U.K. at one- and five-year horizons is calculated from the monthly CPI on all items, as reported by the OECD (see also FRED series GBRCPIALLMINMEI).

Data on U.S. inflation expectations come from FRED and the Survey of Professional Forecasters. From FRED, we use the inflation expectations from the Surveys of Consumers of University of Michigan (series MICH), which covers short-term inflation expectations, and the expected 10-year-ahead inflation implied from Treasury Inflation-Indexed Constant Maturity Securities (series T10YIE). From the Survey of Professional Forecasters, we use the 10-year ahead inflation expectations. These data are shown in Figure 8. Further, we use median one-year-ahead expected inflation from the Survey of Professional Forecasters to construct the deviation of expected inflation from realized inflation, shown in Figure 9.

Growth data come from different sources. In Tables 1–2, the U.S. growth parameter μ is set to match per capita consumption growth, series A794RX0Q048SBEA from FRED Economic Data hosted by the St. Louis Federal Reserve. In Figure 2 and Table 3, we use real per capita GDP growth rates from FRED (series A939RX0Q048SBEA) as the growth rate for the U.S. Average annual growth rates are used, which are computed using December-to-December values. When calibrating to the U.K. data, we use the real GDP growth series from Jordà et al. (2019).

Data on investment and capital stock come from the Bureau of Economic Analysis (BEA) Fixed Assets Accounts Tables. Investment data come from Table 1.5, Line 2 and capital stock data come from Table 1.1, Line 2. In these data, investment as a fraction of capital averaged 7.7% from 1984–2000 and 6.8% from 2001–2021.

Price-dividend ratio data for the U.S. from 1984 to 2021 are from the Center for Research in Security Prices (CRSP). Specifically, we use cum-dividend returns (series VWRETD) and ex-dividend returns (series VWRETX). To calculate the price-dividend ratio, we back out prices and dividends from cum- and ex-dividend returns. This series is plotted in Panel B of Figure 2. We use this procedure to calculate our price-dividend ratio moments for the calibrations in Tables 1 and 2.

For the longer U.S. valuation data, we use prices and dividends on the S&P 500 from Shiller (2000). We also form the cyclically-adjusted price-earnings ratio (CAPE): the price divided by the average inflation-adjusted earnings from the previous 10 years. See (Shiller, 2000) and online data description. For the U.K. valuation data, we use data from Jordà et al. (2019). Jordà et al. aggregate total returns data from Grossman (2002) and from Barclays Equity Gilt Study.

Finally, we obtain the Volatility Index (VIX) series from the Chicago Board Options Exchange (CBOE). The CBOE calculates the risk-neutral expected 30-day quadratic variation using option prices. There are small differences in the calculation methodology over the years; see CBOE white paper.

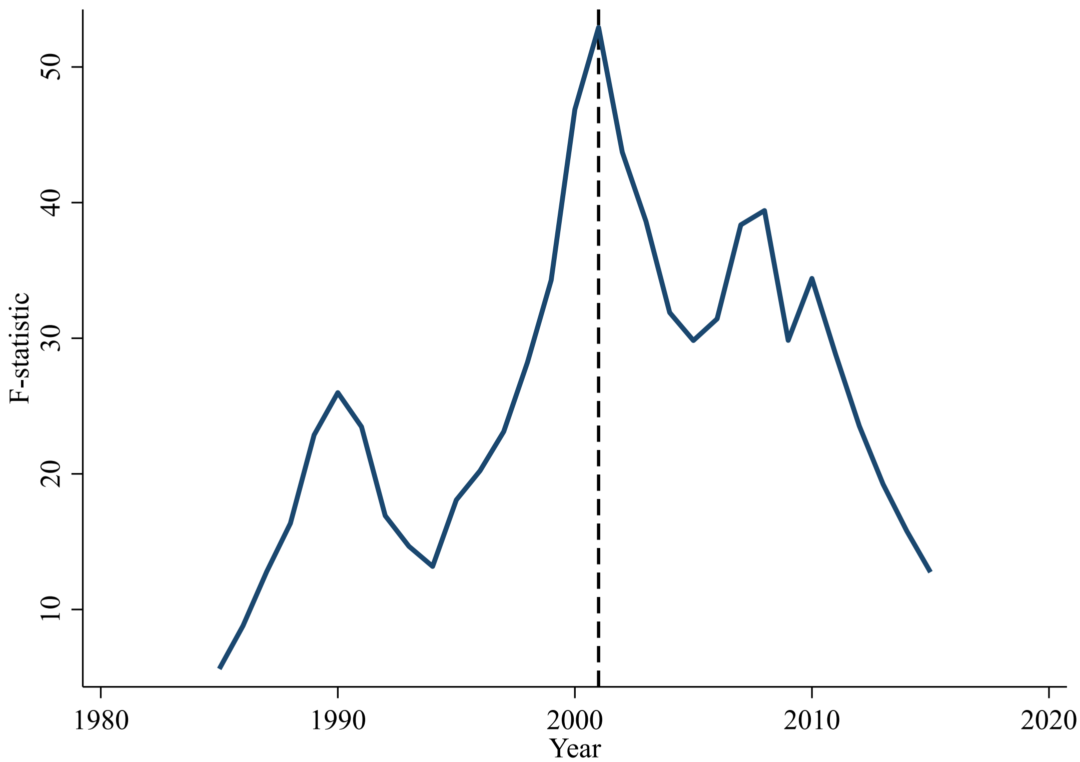
B Structural break test

Throughout the main text, we calibrate the model to data from two subsamples. We determine the most likely date for a structural break in the average one-year inflation-adjusted Treasury yield (denoted y_b in the paper). Specifically, for each potential break year t_{break} since 1985, we estimate the regression[26]

$$y^{\$}_{b,t} - \Delta\pi_t = \beta_0 + \beta_1 \mathbb{1}\{t > t_{\text{break}}\} + \epsilon_t, \tag{B.1}$$

[26] Recall that $y^{\$}_{b,t}$ is the yield on the one-year Treasury bill and $\Delta\pi_t$ is one-year realized inflation, so this difference represents the ex post return on the one-year nominal bond.

Figure B.1: Structural break test on interest rates



Notes: The figure presents F-statistics for the linear regression (B.1), estimated using OLS for all potential break dates from 1985–2015 using data on one-year nominal Treasury bill yields less inflation.

which amounts to estimating y_b separately in each sample period. Figure B.1 plots the F-statistic from this regression as a function of the break point. Evidently, 2001 stands out as the best fit for a structural break in inflation-adjusted yields.

As we mention in the main text, our choice of 2001 as a break date is also consistent with prior work studying secular changes in macroeconomic time series since the 1980s. First, Farhi and Gourio (2018) calibrate their model to two separate data samples around this date. Second, using a regression-based break test very similar to ours, Campbell et al.

(2020) identify 2001 as the most likely year for a structural break in the relation between GDP growth and inflation. They find that the two series were negatively correlated prior to 2001 and became positively correlated thereafter.

C Derivations for Section 3: Endowment economy

C.1 Price-consumption ratio

Given the SDF (3), the Euler equation with respect to the consumption claim is

$$1 = \mathbb{E}_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R^\theta_{W,t+1}\right]. \tag{C.1}$$

Conjecture a constant price-consumption ratio

$$\kappa \equiv (W_t - C_t)/C_t. \tag{C.2}$$

Substituting (C.2) into (C.1) and using $R_{W,t+1} = W_{t+1}/(W_t - C_t)$ implies

$$1 = \beta^\theta \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{\theta(1-\frac{1}{\psi})} \left(\frac{\kappa+1}{\kappa}\right)^\theta\right]. \tag{C.3}$$

Given (2–1),

$$\frac{\kappa}{\kappa+1} = \beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p((1-\eta)^{1-\gamma} - 1)\right]^{\frac{1}{\theta}}. \tag{C.4}$$

A solution exists provided that the right hand side of (C.4) is less than one. We restrict attention to parameter combinations satisfying this restriction. Finally,

$$\kappa = \frac{\beta e^{(1-\frac{1}{\psi})\mu}\left[1+p((1-\eta)^{1-\gamma}-1)\right]^{\frac{1}{\theta}}}{1-\beta e^{(1-\frac{1}{\psi})\mu}\left[1+p((1-\eta)^{1-\gamma}-1)\right]^{\frac{1}{\theta}}}, \tag{C.5}$$

verifying the conjecture.

C.2 Riskfree rate

The riskfree rate is given by the Euler equation for the riskfree asset

$$R_f = \mathbb{E}_t\left[\beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1}\right]^{-1}. \tag{C.6}$$

This simplifies to

$$R_f = \mathbb{E}_t\left[\beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{\kappa}{\kappa+1}\right)^{1-\theta}\right]^{-1}. \tag{C.7}$$

where $\kappa/(\kappa+1)$ is given by (C.4). Solving this yields the expression for the gross riskfree rate

$$R_f = \beta^{-1} e^{\frac{1}{\psi}\mu}\left[1+p((1-\eta)^{-\gamma}-1)\right]^{-1}\left[1+p((1-\eta)^{1-\gamma}-1)\right]^{\frac{\theta-1}{\theta}} \tag{C.8}$$

which implies that the log riskfree rate is given by

$$\log R_f = -\log\beta + \frac{1}{\psi}\mu - \log(1+p((1-\eta)^{-\gamma}-1)) + \left(\frac{\theta-1}{\theta}\right)\log(1+p((1-\eta)^{1-\gamma}-1)). \tag{C.9}$$

C.3 Yield and expected return with sovereign default risk

Consider the defaultable short-term government bond paying $(1 - L_{t+1})$ dollars—that is, 1 dollar in the case of no default and $1 - \lambda\eta$ dollars in the case of default. The price of this claim is obtained by solving the Euler equation

$$Q_t = \mathbb{E}_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1}(1 - L_{t+1})\right], \tag{C.10}$$

which simplifies to

$$Q_t = \mathbb{E}_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{\kappa}{\kappa+1}\right)^{1-\theta} (1 - L_{t+1})\right], \tag{C.11}$$

where $\kappa/(\kappa+1)$ is given by (C.4). This gives the price of the defaultable claim as

$$Q_t = \beta e^{-\frac{1}{\psi}\mu}\left[1 + p((1-\eta)^{1-\gamma} - 1)\right]^{\frac{1-\theta}{\theta}}\left[1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right]. \tag{C.12}$$

The yield on the defaultable claim is defined as $y_{b,t} \equiv -\log Q_t$, and is thus equal to the constant

$$y_b = \log R_f + \log\left(1 + p((1-\eta)^{-\gamma} - 1)\right) - \log\left(1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right), \tag{C.13}$$

where $\log R_f$ is given by (C.9). The expected excess return on the bond is the expected payoff divided by the price, less the log riskfree rate, and therefore equals

$$\begin{aligned}\log \mathbb{E}_t\left[R_{b,t+1}\right] - r_f = {} & \log\left(1 + p((1-\lambda\eta) - 1)\right) \\ & + \log\left(1 + p((1-\eta)^{-\gamma} - 1)\right) - \log\left(1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right). \end{aligned} \tag{C.14}$$

Suppose instead of being subject to outright default, the bond is a nominally riskfree asset and so the government partially defaults through inflation. Assume inflation is given by the process (11). The price of this defaultable claim is obtained by solving the Euler equation

$$Q_t^{\$} = \mathbb{E}_t\left[\beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1}\frac{\Pi_t}{\Pi_{t+1}}\right], \tag{C.15}$$

which simplifies to $Q_t^{\$} = Q_t e^{-\mu_{\pi,t}}$ for the price Q_t given by (C.12). Subsequent results in the main text then follow straightforwardly.

D Volatility Index in a disaster economy

For tractability, we adapt the simple disaster model to continuous time, following Seo and Wachter (2019). Suppose consumption follows the jump-diffusion process

$$\frac{dC_t}{C_{t^-}} = \mu dt + \sigma dB_t + (e^{-z_t} - 1)dN_t, \tag{D.1}$$

where B_t is a standard Brownian motion, N_t is a Poisson process with constant intensity λ, and z_t has time-invariant distribution υ. As in Abel (1999) and Campbell (2003), we model dividends as levered consumption: $D_t = C_t^{\phi}$. Under both power utility and recursive preferences, it follows that the price of the claim to the dividend stream follows the process

$$\frac{dS_t}{S_{t^-}} = \mu_S dt + \phi\sigma dB_t + (e^{-\phi z_t} - 1)dN_t. \tag{D.2}$$

The quadratic variation is then given by

$$QV_{t,t+\tau} \equiv \int_t^{t+\tau} d[\log S, \log S]_s = \phi^2\sigma^2\tau + \int_t^{t+\tau} \phi^2 z_s^2 dN_s. \tag{D.3}$$

For risk-neutral measure Q, the VIX is then given by

$$\mathrm{VIX}_t^2 \equiv \mathbb{E}_t^Q[QV_{t,t+\tau}] = \phi^2 \left(\sigma^2 + \lambda \mathbb{E}_\upsilon \left[e^{\gamma z_t} z_t^2\right]\right) \tau, \tag{D.4}$$

where the last term follows from Girsanov's theorem:

$$\mathbb{E}_{t^-}^Q \left[\phi^2 z_s^2 dN_s\right] = \mathbb{E}_{t^-} \left[\frac{\pi_t}{\pi_{t^-}} \phi^2 z_s^2 dN_s\right] = \lambda \phi^2 \mathbb{E}_\upsilon \left[e^{\gamma z_t} z_t^2\right]. \tag{D.5}$$

Note that these formulas hold for both time-additive utility and recursive preferences.

To calculate the implied VIX in the model, we choose parameters according to our calibration in Table 1: disaster size $z = -\log 0.85$, relative risk aversion coefficient $\gamma = 12$, consumption volatility $\sigma^2 = 0.02$, first sample disaster intensity $\lambda_1 = 0.03$, and second sample disaster intensity $\lambda_2 = 0.07$. These are annualized parameters, so $\tau = 1/12$ matches the time interval used to calculate the VIX. We then choose ϕ^2 such that (D.4) with λ_1 is equal to the empirically observed value 0.2056^2 in the first sample. Given this calibration—which implies $\phi^2 = 19.8$—we calculate that the implied VIX with $\lambda_2 = 0.07$, using this value of ϕ^2, is 23.36 compared to the empirical average of 20.66. Using Newey-West standard errors with two lags on the monthly VIX, the t-statistic on this test is 2.66.

E Production model

E.1 Solution to the no-inventory case

Consider the model in Section 4.1. The agent maximizes (22), subject to (21). Conjecture that

$$V(W_t) = \nu W_t, \tag{E.1}$$

for some constant $\nu > 0$. Substituting this conjecture into (22), with $R_{W,t+1} \equiv R_{f,t+1} + \alpha_t(R_{K,t+1} - R_{f,t+1})$ implies

$$(1-\beta)\log\nu + \log W_t = \max_{C_t,\alpha_t}\left\{(1-\beta)\log C_t + \beta\log\left(W_t - C_t\right) + \frac{\beta}{1-\gamma}\log\left(\mathbb{E}_t\left[R_{W,t+1}^{1-\gamma}\right]\right)\right\}. \tag{E.2}$$

At the optimum. the derivative of the right-hand side with respect to C_t equals zero. Thus:

$$\frac{1-\beta}{C_t} - \frac{\beta}{W_t - C_t} = 0$$

yielding the result $C_t/W_t = 1-\beta$. Setting the derivative of the right hand side with respect to α equal to zero yields (28).

E.2 Solution to the general case

The agent can invest in an inventory asset with net return $r_I = 0$, a riskfree bond with net return $r_{f,t+1}$, and a risky capital asset with net return $r_{K,t+1}$. Let $r_{j,t+1}$, $j \in \mathcal{J} = \{I, f, K\}$, represent net returns, and let $\alpha_{j,t}$ denote the percent allocation of savings to asset j. Note that, in our setting with a binary shock χ_{t+1}, markets are complete, so the agent will be able to construct any state-contingent portfolio return $r_{i,t+1}$. Inventory and capital are the only securities in positive net supply; furthermore, we restrict inventory to be in non-negative supply ($I_t \geq 0$). It follows from this setup that the return on wealth $R_{W,t+1} = \sum_{j\in\mathcal{J}} \alpha_{j,t}(1 + r_{j,t+1})$, where $\sum_{j\in\mathcal{J}} \alpha_{j,t} = 1$.

Suppose that the agent has Epstein-Zin utility with unit EIS. The agent's optimization problem is therefore

$$\max_{C_t,\{\alpha_{j,t}\}_{j\in\mathcal{J}}} \left(C_t^{1-\beta}\left(\mathbb{E}_t\left[V(W_{t+1})^{1-\gamma}\right]\right)^{\frac{\beta}{1-\gamma}}\right), \tag{E.3}$$

subject to the dynamic budget constraint

$$W_{t+1} = (W_t - C_t)R_{W,t+1} = (W_t - C_t)\sum_{j\in\mathcal{J}} \alpha_{j,t}(1 + r_{j,t+1}), \tag{E.4}$$

the portfolio weight restriction

$$\sum_{j\in\mathcal{J}} \alpha_{j,t} = 1, \tag{E.5}$$

and the inventory non-negativity constraint

$$\alpha_{I,t} \geq 0. \tag{E.6}$$

Let ζ_t and ξ_t denote the Lagrange multipliers on the constraints (E.5) and (E.6), respectively.

Substituting (E.1) and the budget constraint (E.4) into (E.3), then taking logs, we again obtain (E.2) and the identical first-order condition for consumption as above. The first-order condition with respect to asset allocation $\alpha_{j,t}$, $j \neq I$, is

$$\beta\mathbb{E}_t\left[R_{W,t+1}^{1-\gamma}\right]^{-1}\mathbb{E}_t\left[R_{W,t+1}^{-\gamma}(1 + r_{j,t+1})\right] = \zeta_t, \tag{E.7}$$

and the first-order condition with respect to the inventory allocation $\alpha_{I,t}$ is

$$\beta\mathbb{E}_t\left[R_{W,t+1}^{1-\gamma}\right]^{-1}\mathbb{E}_t\left[R_{W,t+1}^{-\gamma}\right] + \xi_t = \zeta_t. \tag{E.8}$$

Multiply both sides of (E.7) by $\alpha_{j,t}$, take the sum over $j \in \mathcal{J} \setminus \{I\}$, and substitute in (E.8) to see that

$$\zeta_t = \beta + \xi_t\alpha_{I,t} = \beta, \tag{E.9}$$

by complementary slackness. This implies the Euler equation for gross returns

$$\mathbb{E}_t\left[R_{W,t+1}^{1-\gamma}\right]^{-1}\mathbb{E}_t\left[R_{W,t+1}^{-\gamma}R_{j,t+1}\right] = 1 \tag{E.10}$$

and the Euler equation for inventory

$$\mathbb{E}_t\left[R_{W,t+1}^{1-\gamma}\right]^{-1}\mathbb{E}_t\left[R_{W,t+1}^{-\gamma}\right]+\frac{\xi_t}{\beta}=1. \tag{E.11}$$

Note the market clearing condition $\alpha_{I,t}=1-\alpha_{K,t}$, where $\alpha_{K,t}$ is simply denoted α_t in our setup in the main text. We thus have that $\xi_t>0$ if and only if $\alpha_t<1$.

We now show formally that inventory imposes a zero lower bound. Throughout, we assume that the bond is in zero net supply.

Lemma 1. *If $\alpha_t<1$, then the gross real riskfree rate $R_{f,t+1}=1$. If $\alpha_t=1$, then $R_{f,t+1}\geq 1$ and is equal to the real riskfree rate in a no-inventory economy $R_{f,t+1}^*$.*

Proof. If $\alpha_{I,t}>0$, then $\xi_t=0$ and (E.10) and (E.8) combine to give us $R_{f,t+1}=1$. If $\alpha_{I,t}=0$, then $\xi_t\geq 0$ and

$$R_{f,t+1}=\frac{\beta}{\beta-\xi_t}, \tag{E.12}$$

which is greater than or equal to 1. Moreover, if $\alpha_{I,t}=0$, then market clearing implies $R_{W,t+1}=R_{K,t+1}$ and the Euler equation (E.10) yields

$$R_{f,t+1}=\mathbb{E}_t\left[R_{K,t+1}^{1-\gamma}\right]\mathbb{E}_t\left[R_{K,t+1}^{-\gamma}\right]^{-1}, \tag{E.13}$$

which is the same as the riskfree rate $R_{f,t+1}^*$ in the no-inventory economy. ■

We next show that the unconstrained riskfree rate determines α.

Theorem 1. *If the unconstrained gross riskfree rate $R_{f,t+1}^*<1$, then $\alpha_t<1$ and the constrained riskfree rate $R_{f,t+1}=1$. If $R_{f,t+1}^*\geq 1$, then $\alpha_t=1$ and the equilibrium is as in a standard no-inventory production economy with $R_{f,t+1}=R_{f,t+1}^*$.*

Proof. We will prove the theorem by contradiction using Lemma 1.

Suppose $R^*_{f,t+1} < 1$ and $\alpha_{I,t} = 0$. Then $R_{f,t+1} = R^*_{f,t+1} < 1$, which contradicts Lemma 1. It must therefore be the case that $R^*_{f,t+1} < 1$ implies $\alpha_{I,t} > 0$, which implies $R_{f,t+1} = 1$.

Now suppose $R^*_{f,t+1} > 1$ and $\alpha_{I,t} > 0$. Then $R_{f,t+1} = 1 < R^*_{f,t+1}$, which contradicts Lemma 1. Moreover, in the knife-edge case $R^*_{f,t+1} = 1$, the equilibrium conditions (E.10) and (E.8) imply $\xi_t = 0$, which implies that $\alpha_{I,t} = 0$ and $R_{f,t+1} = R^*_{f,t+1} = 1$. Thus, it must be that $R^*_{f,t+1} \geq 1$ implies $\alpha_{I,t} = 0$, which implies $R_{f,t+1} = R^*_{f,t+1} \geq 1$. ■

We conjecture that the price-dividend ratio depends only on the current state χ_t (i.e., whether the disaster occurred or not). The intuition for this is that output growth Y_{t+1}/Y_t is a function of χ_t only. Thus,

$$1 = \mathbb{E}_t \left[R^{1-\gamma}_{W,t+1}\right]^{-1} \mathbb{E}_t \left[R^{-\gamma}_{W,t+1} \left(\frac{\kappa^Y(\chi_{t+1}) + 1}{\kappa^Y(\chi_t)}\right)\left(\frac{Y_{t+1}}{Y_t}\right)\right]. \tag{E.14}$$

This implies that we have two equations, one for the non-disaster state,

$$\begin{aligned}\kappa^Y(0) = \hat{\beta}\Big[&(1-p)(1+\alpha r_{K,0})^{1-\gamma}(\kappa^Y(0)+1) \\ &+ p(1+\alpha r_{K,\eta})^{-\gamma}(\kappa^Y(\eta)+1)(1-\eta)(1+\alpha r_{K,0})\Big],\end{aligned} \tag{E.15}$$

and one for the disaster state,

$$\begin{aligned}\kappa^Y(\eta) = \hat{\beta}\Big[&(1-p)(1+\alpha r_{K,0})^{-\gamma}(\kappa^Y(0)+1)(1-\eta)^{-1}(1+\alpha r_{K,\eta}) \\ &+ p(1+\alpha r_{K,\eta})^{1-\gamma}(\kappa^Y(\eta)+1)\Big].\end{aligned} \tag{E.16}$$

In these equations, $\hat{\beta} \equiv \beta\left[(1-p)(1+\alpha r_{K,0})^{1-\gamma} + p(1+\alpha r_{K,\eta})^{1-\gamma}\right]^{-1}$, $r_{K,0} \equiv (1-\delta+A)-1$, and $r_{K,\eta} \equiv (1-\delta+A)(1-\eta)-1$. The solution to this system is as stated in the main text (after defining the weights ν).

Although the price-dividend ratio is state-dependent when the agent chooses to hold inventory, the risk premium is not. The risk premium at time t when the agent holds inventory is given by $\log \mathbb{E}_t[R^Y_{t+1}] - \log R_f$, for the expected return on the output claim

$$\mathbb{E}_t[R_{Y,t+1}] = \mathbb{E}_t\left[\left(\frac{\kappa^Y(\chi_{t+1})+1}{\kappa^Y(\chi_t)}\right)\left(\frac{Y_{t+1}}{Y_t}\right)\right]. \tag{E.17}$$

If the expected return on the output claim is the same across states, then so is the risk premium. In the no-disaster state, the expected return on the output claim is

$$\mathbb{E}_t[R_{Y,t+1}|\chi_t = 0] = \left(\frac{(1-p)\kappa^Y(0) + p\kappa^Y(\eta) + 1}{\kappa^Y(0)}\right)\times \\ \Big(\beta(1-p\eta)\left(\alpha(1-\delta+A)+1-\alpha\right)\Big) \tag{E.18}$$

and in the disaster state by

$$\mathbb{E}_t[R_{Y,t+1}|\chi_t = \eta] = \left(\frac{(1-p)\kappa^Y(0) + p\kappa^Y(\eta) + 1}{\kappa^Y(\eta)}\right)\times \\ \left(\beta(1-p\eta)\left(\alpha(1-\delta+A)+\left(\frac{1-\alpha}{1-\eta}\right)\right)\right). \tag{E.19}$$

Examining the two expressions, we see that the expected return in both states are the same if and only if

$$\kappa^Y(\eta)(1-\eta)\Big(\alpha(1-\delta+A)+1-\alpha\Big) = \kappa^Y(0)\Big(\alpha(1-\delta+A)(1-\eta)+1-\alpha\Big).$$

The terms inside the parentheses can be written so that

$$\kappa^Y(\eta)(1-\eta)(1+\alpha r_{K,0}) = \kappa^Y(0)(1+\alpha r_{K,\eta}),$$

which is true if we substitute in the expressions for $\kappa^Y(\chi_t)$. This implies that, while the price-dividend ratio is time-varying, the risk premium is not.