INTERNET APPENDIX

A Interest-rate sensitivity: Definitions and derivations

A.1 Interest-rate sensitivity of an asset

Consider a claim to a stream of cashflows $\{D_{t+k} : k \in \{1, 2, \dots\}\}$. Let P_t denote the price of the asset and $\{R_{t+k} : k \in \{1, 2, \dots\}\}$ its future one-period returns (i.e., $R_{t+k} = (P_{t+k} + D_{t+k})/P_{t+k-1}$). The *interest-rate sensitivity* of an asset is defined as the elasticity of its price with respect to the interest rate,

$$\varepsilon_r(P_t) \equiv -\frac{\partial \log P_t}{\partial r_{ft}}, \tag{A.1}$$

holding fixed the distributions of future cashflows and future returns in excess of the riskfree rate.

Now let us derive an expression for interest-rate sensitivity under the assumption that interest rates follow an AR(1). It will be easiest to do this if we divide the asset into separate claims to the cashflows at each horizon k (the "strips"). Letting P_{kt} denote the price of the claim to the single cashflow D_{t+k}, we have $P_t = \sum_{k=1}^{\infty} P_{kt}$. The interest-rate sensitivity of the asset is then the value-weighted average of the sensitivities of each strip:

$$\varepsilon_r(P_t) = \sum_{k=1}^{\infty} \frac{P_{kt}}{P_t} \varepsilon_r(P_{kt}). \tag{A.2}$$

To derive these elasticities, note that, from the definition of a return,

$$P_{kt} = \frac{P_{k-1,t+1}}{R_{k,t+1}} = \left(\prod_{k'=1}^{k} R_{k',t+1+k-k'}^{-1} \right) D_{t+k}. \tag{A.3}$$

Thus, defining the excess return $\hat{r}_{k',t+k} \equiv \log(R_{k',t+k}/R_{f,t+k-1})$ and taking logs then expectations,

$$\log P_{kt} = -\sum_{k'=1}^{k} (\mathbb{E}_t[r_{f,t+k-k'}] + \mathbb{E}_t[\hat{r}_{k',t+1+k-k'}]) + \mathbb{E}_t[\log D_{t+k}]. \tag{A.4}$$

Now, using the fact that $\mathbb{E}_t[r_{f,t+k}] = \bar{r}_f + \varphi^k(r_{ft} - \bar{r}_f)$, we have that

$$\varepsilon_r(P_{kt}) = -\frac{\partial \log P_{kt}}{\partial r_{ft}} = \sum_{k'=1}^{k} \left(\varphi^{k-k'} + \frac{\partial \mathbb{E}_t[\hat{r}_{k',t+1+k-k'}]}{\partial r_{ft}} \right) - \frac{\partial \mathbb{E}_t[\log D_{t+k}]}{\partial r_{ft}} = \frac{1 - \varphi^k}{1 - \varphi}. \tag{A.5}$$

The last equality follows from the assumption that expected excess returns and cashflow growth are held constant, so $\partial \mathbb{E}_t[\hat{r}_{t+k'}]/\partial r_{ft} = \partial \mathbb{E}_t[D_{t+k}]/\partial r_{ft} = 0$. Substituting this into

the expression above, we have

$$\varepsilon_r(P_t) = \sum_{k=1}^{\infty} \frac{P_{kt}}{P_t} \frac{1-\varphi^k}{1-\varphi}. \tag{A.6}$$

A.2 Relating interest-rate sensitivity to duration

The (Macaulay) *duration* of an asset is defined as the value-weighted timing of its cash-flows:

$$\text{dur}(P_t) \equiv \sum_{k=1}^{\infty} \frac{P_{kt}}{P_t} k, \tag{A.7}$$

where the strip prices P_{kt} are defined above.

Notice that $\lim_{\varphi \to 1} \varepsilon_r(P_t) = \text{dur}(P_t)$, meaning that the duration is the same as the response to a permanent change in interest rates. Our empirical measure of interest-rate sensitivity is derived from a first-order approximation of ε_r around the duration. To see this, note that, to a first-order, $\varphi^k \approx \varphi^{\text{dur}(P_t)} + \varphi^{\text{dur}(P_t)} \log \varphi (k - \text{dur}(P_t))$. Substituting this approximation back into the expression for ε_r gives

$$\varepsilon_r(P_t) = \frac{1 - \sum_{k=1}^{\infty} \frac{P_{kt}}{P_t} \varphi^k}{1-\varphi} \approx \frac{1-\varphi^{\text{dur}(P_t)}}{1-\varphi}. \tag{A.8}$$

A.3 Interest-rate sensitivity of wealth

Just as the rate sensitivity of an asset is the value-weighted average of the sensitivities of its strips, the rate sensitivity of wealth is the value-weighted average of the sensitivities of its assets. Suppose a household has wealth $W_t = \sum_j A_{jt} - \sum_k D_{kt}$. In the case of positive net worth $W_t > 0$, it is clear that

$$\varepsilon_r(W_t) = -\sum_j \frac{A_{jt}}{W_t} \frac{\partial \log A_{jt}}{\partial r_{ft}} + \sum_k \frac{D_{kt}}{W_t} \frac{\partial \log D_{kt}}{\partial r_{ft}} \tag{A.9}$$

$$= \sum_j \frac{A_{jt}}{W_t} \varepsilon_r(A_{jt}) - \sum_k \frac{D_{kt}}{W_t} \varepsilon_r(D_{kt}). \tag{A.10}$$

The definition of rate sensitivity above applies only to positive-wealth households. This is not an issue in our empirics, as we only include these households in our computations. Even so, if one wanted to measure rate sensitivity for a household with negative wealth, a simple generalization that handles this case would be

$$\varepsilon_r(W_t) \equiv -\frac{\partial W_t}{\partial r_{ft}} \frac{1}{|W_t|}. \tag{A.11}$$

For $W_t > 0$, this reverts to the original definition (by the chain rule). For a household with negative networth,

$$\varepsilon_r(W_t) = \sum_j \frac{A_{jt}}{|W_t|}\varepsilon_r(A_{jt}) - \sum_k \frac{D_{kt}}{|W_t|}\varepsilon_r(D_{kt}). \tag{A.12}$$

The absolute value in the denominator is necessary for getting the sign of the elasticity right. For example, suppose a household has \$100 of assets and \$200 of debt, both with the same rate sensitivity (say, 2). If rates fall by 1%, networth should fall by roughly \$2, from $-$\$100 to about $-$\$102.[33] In this case, (A.12) implies a rate sensitivity of wealth of -2, reflecting the fact that a decline in rates has a negative effect on wealth.

B Data appendix

This appendix adds the measurement details that Section 1.3 leaves out. The Supplementary Codebook documents the rest: the SCF variables behind each balance-sheet component, every imputation and its parameters, the source series, and the script that writes each file.

B.1 Survey of Consumer Finances

We use the eleven SCF waves from 1989 to 2019 and all five implicates of each. We combine the summary extract with the full public data, which report the loan terms, vehicle ages, and business details that the extract omits. We construct networth as

$$\begin{aligned}\texttt{networth_d} = \texttt{cash_dep} + \texttt{equity} + \texttt{fixed_inc} + \texttt{real_estate} \\ + \texttt{bus} + \texttt{vehic} - \texttt{mortgage_dbt} - \texttt{vehic_dbt} - \texttt{edu_dbt} - \texttt{other_dbt},\end{aligned}$$

where each component follows the definitions of the SCF summary extract; the codebook lists the variables in each. We also use household wage income (`wageinc`), which we divide by the number of adults in the household and by the Social Security wage index so that it is comparable over time. We count married and widowed respondents as two-adult households and all others as single-adult households.

B.2 Duration component calculations

Heterogeneity across the wealth distribution. For equity, fixed income, and private business, we let duration vary across six wealth groups: the bottom 50%, 50–90%, 90–99%, 99–99.9%, 99.9–99.99%, and top 0.01%. For each group g, we compute a valuation multiple m_g from SCF holdings and income, deflated by the Social Security wage index: equity

[33] Under a rate sensitivity of 2, assets will rise from \$100 to \$102 and debt will rise from \$200 to \$204.

over dividend income, bonds over interest income, and business value over business income. We then scale the aggregate duration of the asset by the mean-preserving factor

$$\alpha_g = \frac{m_g}{\bar{m}}, \qquad \bar{m} = \frac{\sum_g A_g\, m_g}{\sum_g A_g},$$

where A_g is the group's holdings of the asset, so that $\sum_g A_g\, \alpha_g / \sum_g A_g = 1$. The aggregate durations are the stock-market estimates of Greenwald et al. (2025) for equity, the Bloomberg index duration of each bond type for fixed income, and the annual valuation ratio below for private business.

Private business wealth. Business income in the SCF mixes dividends with pay for the owner's work, so we subtract an estimate of that pay. If the respondent manages one of the household's businesses and reports being self-employed without a salary, we subtract a predicted wage. If the respondent manages a business and either draws a salary or works for someone else, we add the reported wage to business income and subtract the larger of the reported and predicted wage. We treat spouses the same way, and all other owners keep their full business income. The predicted wage comes from a weighted regression of each adult's wage income, scaled by the wage index, on a cubic in age interacted with race, education, and sex, estimated on earners aged 18 to 62. Summing business value and the resulting cashflows within each survey year then gives (5).

Defined benefit pensions. We take DB pension entitlements by wealth group from the Federal Reserve's Distributional Financial Accounts and keep the funded share, the ratio of pension fund assets to entitlements in Table L.117 of the Flow of Funds, as Saez and Zucman (2016) do. Within each wealth group and year, we allocate the funded total to SCF households in proportion to

$$w_{it} = \text{accrual}_{it} \times \text{mortality}_{it} \times \text{income}_{it} \times \mathbf{1}[\text{has DB}_{it}].$$

Here accrual is the predicted share of lifetime benefits already earned, from a regression of the accrued share of Social Security benefits on income and wealth centiles, age, and sex; it is one after age 65 and zero before age 20. Mortality is remaining life expectancy relative to that at age 65, capped at one. Income is wage income relative to the group mean, capped at five. The indicator equals one if the respondent or spouse reports a DB plan from a current or past job. We split each household's pension across cash deposits, fixed income, and equity in the proportions that private pension funds hold in Table L.117. The equity

piece receives the duration of the stock market and the fixed-income piece that of the US aggregate bond index.

Debts. Each loan's duration follows from equation (6) and its time remaining. For the loans that the SCF reports in detail, time remaining comes from the origination year and the original term. When these are missing or imply a negative term, we solve for the term of a fixed-payment loan from the balance B, the initial amount L, the gross interest rate R, and the years since origination p:

$$T = \frac{\log(R^p - B/L) - \log(1 - B/L)}{\log R} - p.$$

Aggregated additional loans and lines of credit report only a balance, a payment X, and at most a rate, so we set $T = -\log(1 - B(R-1)/X)/\log R$, using the average rate on primary loans of the same type where the rate is missing. Loans with less than a year left receive a duration of one, loans without enough information receive the median duration, and credit card balances receive a duration of one. Adjustable-rate mortgages are capped at four years, the average time until their rate floats in the SCF. Student loans are valued at the present value of payments under income-driven repayment, following Catherine and Yannelis (2023): borrowers pay 10% of earnings above a threshold for up to 25 years.

Social Security wealth. The interest-rate sensitivity of Social Security wealth comes from the methodology of Catherine et al. (2025). We compute their baseline risk-adjusted Social Security wealth under the net present value concept and the Treasury yield curve. We then recompute it with the log forward rate at horizon h in survey year t raised to $\tilde{f}_{h,t} = f_{h,t} + \varphi^h 0.01$, where φ is the calibrated persistence in Table 1. This is the same as a one-period shock of 0.01 to the log riskfree rate under the process in equation (3).

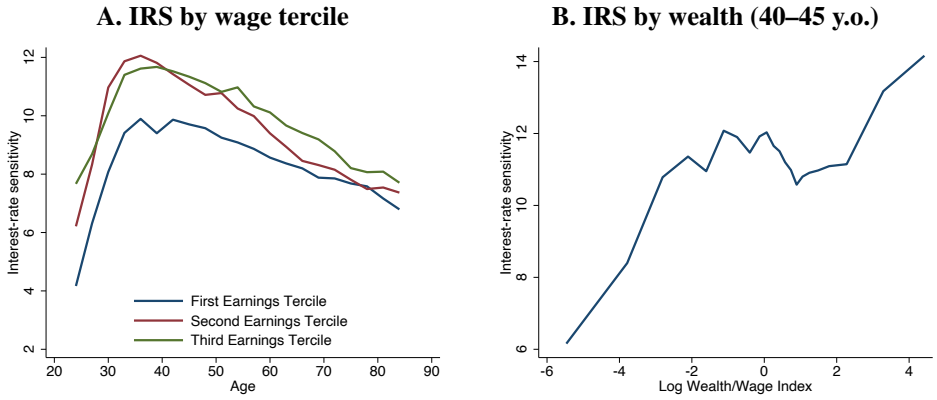
B.3 Evidence from Europe

We repeat the measurement for the European Union with all four waves of the Household Finance and Consumption Survey of the European Central Bank (2010, 2014, 2017, and 2021), all participating countries, and all five implicates. Each asset receives the interest-rate sensitivity of the same asset in the US, using 2019 US estimates for the 2021 wave.[34] We split other financial assets and voluntary pensions, which the survey does not break

[34]The survey does not report bond types or vehicle ages, so all fixed income receives a duration of 5.64 years and all vehicles 6 years. Equity and private business receive aggregate US durations, without the scaling across wealth groups that we apply in the US.

down, evenly between equity and fixed income. Debts follow Appendix B.2, with two differences: adjustable-rate mortgages receive a duration of one year, the typical reset period in Europe, and other non-mortgage loans receive the average duration of mortgage and property loans. We winsorize the rate sensitivity of net worth at the 1st and 99th percentiles. Figure B.1 shows the results. As in the US, interest-rate sensitivity rises with income and wealth and is hump-shaped over the life cycle.

Figure B.1: Interest-rate sensitivity in the European Union



While the broad trends are similar, the relationship between income and wealth and interest-rate sensitivity is less stark in the EU, particularly at the top of the distribution. A potential reason for this is illustrated by Figure B.2, which shows that countries in the EU give more generous benefits to higher earners. Indeed, in the US, essentially no one receives more than 60% of average income, whereas in the EU it is not uncommon for an individual to receive more than 100% of the national average wage. This is what our model predicts: countries that replace more income for high earners should have a flatter relationship between income, wealth, and the interest-rate sensitivity of marketable wealth.

C Model derivations

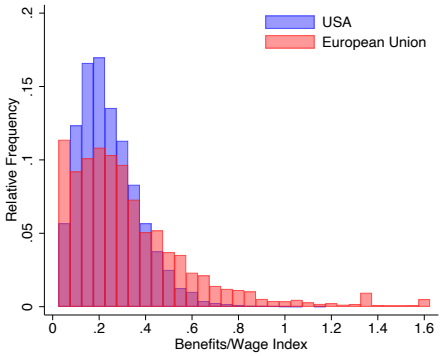
C.1 Long-term bond returns

This section explains how the riskfree rate dynamics (3) imply the n-period bond returns (10). Since it has no intermediate cashflows, the bond's return from t to $t+1$ is

$$R_{n,t+1} = \frac{P_{n-1,t+1}}{P_{nt}} \equiv \frac{e^{-(n-1)y_{n-1,t+1}}}{e^{-ny_{nt}}}, \tag{C.1}$$

where the yield y_{nt}, under the expectations hypothesis, is given by

Figure B.2: Generosity of public pensions: US vs EU



$$y_{nt} \equiv \frac{1}{n}\log\left(\frac{1}{P_{nt}}\right) = \frac{1}{n}\sum_{j=1}^{n}(\mathbb{E}_t r_{f,t+j-1} + \mu_j). \tag{C.2}$$

Moreover, note that (3) iterates forward to the expression

$$r_{f,t+j} = (1-\varphi^j)\bar{r}_f + \varphi^j r_{ft} + \sum_{k=1}^{j}\varphi^{j-k}\sigma_r\epsilon_{r,t+k}. \tag{C.3}$$

Substituting the riskfree rates (C.3) into the yield expression (C.2) and evaluating expectations implies

$$y_{nt} = \bar{r}_f + \frac{1}{n}\frac{1-\varphi^n}{1-\varphi}(r_{ft} - \bar{r}_f) + \frac{1}{n}\sum_{j=1}^{n}\mu_j. \tag{C.4}$$

Taking logs of (C.1) and substituting (C.4) into the yield then implies the log return

$$\begin{aligned} r_{n,t+1} &= n y_{nt} - (n-1)y_{n-1,t+1} && \text{(C.5)}\\ &= \bar{r}_f + \frac{1-\varphi^n}{1-\varphi}(r_{ft} - \bar{r}_f) - \frac{1-\varphi^{n-1}}{1-\varphi}(r_{f,t+1} - \bar{r}_f) + \mu_n && \text{(C.6)}\\ &= r_{ft} + \mu_n - \frac{1-\varphi^{n-1}}{1-\varphi}\sigma_r\epsilon_{r,t+1}, && \text{(C.7)} \end{aligned}$$

the stated expression (10).

C.2 Stock returns

Let D_t denote the dividends of the market and V_t its value. We make two assumptions. First, expected log returns equal the riskfree rate plus a constant risk premium: $\mathbb{E}_t[r_{s,t+1}] = r_{ft} + \mu_s$. Second, since dividends are cointegrated with aggregate income, we follow Longstaff and Piazzesi (2004) and define the log dividend-income ratio $dl_t \equiv \log(D_t/L_t)$ by the autoregression $dl_{t+1} = (1-\varphi_d)\overline{dl} + \varphi_d dl_t + \epsilon_{d,t+1}$, for some shock $\epsilon_{d,t+1}$ with $\mathbb{E}_t[\epsilon_{d,t+j}] = 0$. This implies the log dividend growth rate

$$g_{D,t+1} = g_{t+1} + \Delta dl_{t+1} = \lambda_{gr} r_{f,t+1} + g_{1,t+1} - (1-\varphi_d)(dl_t - \overline{dl}) + \epsilon_{d,t+1}. \tag{C.8}$$

Next, we linearize the stock return under these assumptions. By definition, the return on the stock market is $R_{s,t+1} = \frac{D_{t+1}+V_{t+1}}{V_t} = \frac{D_{t+1}}{D_t}\left(\frac{1+V_{t+1}/D_{t+1}}{V_t/D_t}\right)$. Now take logs and linearize to get the Campbell-Shiller decomposition. Letting $g_{D,t+1} = \log(D_{t+1}/D_t)$ and $vd_t = \log(V_t/D_t)$ represent the log of dividend growth and of the price-dividend ratio, respectively, we have $r_{s,t+1} \approx g_{D,t+1} + \kappa_0 + \kappa_1 vd_{t+1} - vd_t$, where

$$\kappa_1 \equiv \frac{1}{1+\exp\{-\overline{vd}\}} \quad \text{and} \quad \kappa_0 \equiv -\log\kappa_1 + (1-\kappa_1)\overline{vd}, \tag{C.9}$$

for $\overline{vd} = \mathbb{E}[vd_t]$. Solving the linearized equation forward for the price-dividend ratio implies

$$vd_t = g_{D,t+1} - r_{s,t+1} + \kappa_0 + \kappa_1 vd_{t+1} = \frac{\kappa_0}{1-\kappa_1} + \sum_{j=0}^{\infty}\kappa_1^j(g_{D,t+1+j} - r_{s,t+1+j}), \tag{C.10}$$

which holds both in realization and in expectation.

We know the expected stock return $\mathbb{E}_t[r_{s,t+1}]$ by assumption. Using the linearization, the unexpected component of the stock return equals $r_{s,t+1} - \mathbb{E}_t[r_{s,t+1}] = g_{D,t+1} - \mathbb{E}_t[g_{D,t+1}] + \kappa_1(vd_{t+1} - \mathbb{E}_t[vd_{t+1}])$. Because risk premia are constant, the unexpected change in the price-dividend ratio equals:

$$\begin{aligned}\kappa_1(vd_{t+1} - \mathbb{E}_t[vd_{t+1}]) = &\sum_{j=1}^{\infty}\kappa_1^j(\mathbb{E}_{t+1}[g_{D,t+1+j}] - \mathbb{E}_t[g_{D,t+1+j}]) \\ &- \sum_{j=1}^{\infty}\kappa_1^j(\mathbb{E}_{t+1}[r_{f,t+j}] - \mathbb{E}_t[r_{f,t+j}]),\end{aligned} \tag{C.11}$$

and hence

$$r_{s,t+1} - \mathbb{E}_t[r_{s,t+1}] = \sum_{j=0}^{\infty}\kappa_1^j(\mathbb{E}_{t+1}[g_{D,t+1+j}] - \mathbb{E}_t[g_{D,t+1+j}]) - \sum_{j=1}^{\infty}\kappa_1^j(\mathbb{E}_{t+1}[r_{f,t+j}] - \mathbb{E}_t[r_{f,t+j}]). \tag{C.12}$$

Note first that $\mathbb{E}_{t+1}[r_{f,t+j}] - \mathbb{E}_t[r_{f,t+j}] = \varphi^{j-1}\sigma_r\epsilon_{r,t+1}$, and therefore that

$$\sum_{j=1}^{\infty}\kappa_1^j(\mathbb{E}_{t+1}[r_{f,t+j}] - \mathbb{E}_t[r_{f,t+j}]) = \frac{\kappa_1}{1-\kappa_1\varphi}\sigma_r\epsilon_{r,t+1}. \tag{C.13}$$

Next, note that

$$\begin{aligned}\mathbb{E}_{t+1}[g_{D,t+1+j}] - \mathbb{E}_t[g_{D,t+1+j}] &= \lambda_{gr}(\mathbb{E}_{t+1}[r_{f,t+1+j}] - \mathbb{E}_t[r_{f,t+1+j}]) \\ &+ (\mathbb{E}_{t+1}[g_{1,t+1+j}] - \mathbb{E}_t[g_{1,t+1+j}]) - (1-\varphi_d)(\mathbb{E}_{t+1}[dl_{t+j}] - \mathbb{E}_t[dl_{t+j}]) + \mathbb{I}_{j=0}\epsilon_{d,t+1}.\end{aligned} \tag{C.14}$$

The riskfree rate component equals

$$\lambda_{gr}(\mathbb{E}_{t+1}[r_{f,t+1+j}] - \mathbb{E}_t[r_{f,t+1+j}]) = \lambda_{gr}\varphi^j(r_{f,t+1} - \mathbb{E}_t[r_{f,t+1}]) = \lambda_{gr}\varphi^j\sigma_r\epsilon_{r,t+1}. \tag{C.15}$$

Similarly,

$$\mathbb{E}_{t+1}[g_{1,t+1+j}] - \mathbb{E}_t[g_{1,t+1+j}] = \varphi_g^j(g_{1,t+1} - \mathbb{E}_t[g_{1,t+1}]) = \varphi_g^j \epsilon_{g,t+1}, \tag{C.16}$$

and $(1-\varphi_d)(\mathbb{E}_{t+1}[dl_{t+j}] - \mathbb{E}_t[dl_{t+j}]) = (1-\varphi_d)\varphi_d^{j-1}\epsilon_{d,t+1}$. Consequently,

$$\sum_{j=0}^{\infty} \kappa_1^j(\mathbb{E}_{t+1}[g_{D,t+1+j}] - \mathbb{E}_t[g_{D,t+1+j}]) = \frac{\lambda_{gr}\sigma_r}{1-\kappa_1\varphi}\epsilon_{r,t+1} + \frac{1}{1-\kappa_1\varphi_g}\epsilon_{g,t+1} + \frac{1-\kappa_1}{1-\kappa_1\varphi_d}\epsilon_{d,t+1}. \tag{C.17}$$

Putting all of this together, the unexpected stock return equals

$$r_{s,t+1} - \mathbb{E}_t[r_{s,t+1}] = \frac{\lambda_{gr}-\kappa_1}{1-\kappa_1\varphi}\sigma_r\epsilon_{r,t+1} + \frac{1}{1-\kappa_1\varphi_g}\epsilon_{g,t+1} + \frac{1-\kappa_1}{1-\kappa_1\varphi_d}\epsilon_{d,t+1}. \tag{C.18}$$

Substituting in the expected return, we have an expression for the return.

Finally, to get to the expression (12) in the main text, we assume that the shock ϵ_d can be decomposed such that $\frac{1-\kappa_1}{1-\kappa_1\varphi_d}\epsilon_{d,t+1} = \lambda_{dg}\epsilon_{g,t+1} + \epsilon_{s,t+1}$, where ϵ_g is the growth shock and ϵ_s is the market-specific shock. For the loading λ_{sr} on interest-rate shocks in (12), we use the fact that, under this linearization, the interest-rate sensitivity of the stock equals

$$\frac{\kappa_1}{1-\kappa_1\varphi} = \frac{\sigma_{n_s}}{\sigma_r} = \frac{1-\varphi^{n_s-1}}{1-\varphi} \implies \kappa_1 = \frac{\sigma_{n_s}/\sigma_r}{1+\varphi\sigma_{n_s}/\sigma_r} = \frac{1-\varphi^{n_s-1}}{1-\varphi^{n_s}}, \tag{C.19}$$

where n_s is the duration of the stock market. We then use this to compute the loading on the interest rate,

$$\lambda_{sr} = \frac{\kappa_1 - \lambda_{gr}}{1-\kappa_1\varphi}. \tag{C.20}$$

Together, this gives the stock return (12).

C.3 Housing returns

To derive housing returns, we need to make assumptions about depreciation and maintenance of the housing stock, as well as transaction costs. First, we assume that the housing stock depreciates each period and owners offset this by investing a proportionate maintenance cost. Second, we assume that homeowners incur proportionate transaction costs, representing the cost of moving. In the model, it is costless for agents to change house size, and thus households move houses every period and pay this transaction cost each period.

These assumptions manifest themselves in the housing return, which equals $R_{hi,t+1} = \frac{P_{n_h-1,t+1}}{P_{n_h t}}\frac{P_{h,t+1}}{P_{ht}}\frac{\tilde{P}_{i,t+1}}{\tilde{P}_{it}}$, as follows. The bond component of the return accounts for the fact that the house depreciates over the period, so its duration falls from n_h to $n_h - 1$, as would a long-term bond. The drift μ_h in (14) includes not only common price appreciation, but also nets out the costs to the homeowner of transacting and of maintaining the property. The investment in maintenance offsets the depreciation, and in doing so returns the bond

portion of the house back to its original duration n_h for the new set of homebuyers. And finally, because the house is sold after this return is realized, the idiosyncratic house price shock only accumulates over the one period, at which point the homeowner sells the house and buys a new one at the regional price P_{ht}.

C.4 Private business valuation

Here we derive the valuation of private business at exit (32). Both in our model and in empirical measures of the value of the business, the relevant valuation is under the discount rate and cashflow dynamics after the company is sold. For discount rates, we assume that the business is sold to diversified investors—this will be the case, for instance, if the company goes public. Because the new owners will be diversified, they will not demand compensation for idiosyncratic risk, so the relevant risk premium will be the same as that of the market. Future cashflows L_i can follow a general process on which we impose only one fairly minor assumption: growth in the persistent part of the idiosyncratic component of income is not predictable from current information. Formally, for all $j \geq 0$, $\mathbb{E}_t[\tilde{L}_{i,t+1+j}/\tilde{L}_{i,t+j}] = \mathbb{E}[\tilde{L}_{i,t+1+j}/\tilde{L}_{i,t+j}]$. Note that this assumption allows for the possibility that the cashflow growth dynamics could change after exit. The only purpose of this assumption is to simplify the model; it affects fluctuations in valuation over time, but not the average level of valuations, as we will see from what follows.

Let E_{it} denote the present value of future earnings $\{L_{i,t+j}\}$. Let e_{it} and $l_{i,t+j}$ denote logs of these variables and $el_{it} \equiv e_{it} - l_{it}$ the current valuation ratio. In what follows, we will suppress i subscripts. Applying the same Campbell-Shiller linearization as we did for the stock market in Appendix C.2, we have that the one-period log return on the business is $r_{e,t+1} \approx g_{L_i,t+1} + \kappa_{E0} + \kappa_E el_{t+1} - el_t$, where

$$\kappa_E \equiv \frac{1}{1+\exp\{-\overline{el}\}} \quad \text{and} \quad \kappa_{E0} \equiv -\log \kappa_E + (1-\kappa_E)\overline{el}, \tag{C.21}$$

and $\overline{el} \equiv \mathbb{E}[el_t]$ is the unconditional mean valuation ratio. Iterating forward and taking expectations, we have

$$el_t - \overline{el} = \sum_{j=0}^{\infty} \kappa_E^j (\mathbb{E}_t[g_{l,t+1+j}] - \bar{g}_l) - \sum_{j=0}^{\infty} \kappa_E^j (\mathbb{E}_t[r_{e,t+1+j}] - \bar{r}_e). \tag{C.22}$$

Because the firm is sold to diversified investors, it has the same expected return as the stock market, so

$$\mathbb{E}_t[r_{e,t+1+j}] = \mathbb{E}_t[r_{s,t+1+j}] = \mathbb{E}_t[r_{f,t+j}] + \mu_s = (1-\varphi^j)\bar{r}_f + \varphi^j r_{ft} + \mu_s. \tag{C.23}$$

This implies

$$\sum_{j=0}^{\infty} \kappa_E^j (\mathbb{E}_t[r_{e,t+1+j}] - \bar{r}_e) = \frac{1}{1-\kappa_E \varphi}(r_{ft} - \bar{r}_f). \tag{C.24}$$

The cashflow growth component can be further divided into aggregate wage index growth and idiosyncratic growth. The aggregate growth component is

$$\sum_{j=0}^{\infty} \kappa_E^j (\mathbb{E}_t[g_{1,t+1+j}] - \bar{g}_1) + \frac{\lambda_{gr}}{1-\kappa_E \varphi}(r_{ft} - \bar{r}_f) = \frac{1}{1-\kappa_E \varphi_g}(g_{1t} - \bar{g}_1) + \frac{\lambda_{gr}}{1-\kappa_E \varphi}(r_{ft} - \bar{r}_f). \tag{C.25}$$

And we have assumed that the idiosyncratic component is not predictable, so

$$\sum_{j=0}^{\infty} \kappa_E^j (\mathbb{E}_t[g_{\tilde{l},t+1+j}] - \mathbb{E}[g_{\tilde{l},t+1+j}]) = 0.$$

Consequently,

$$el_t = \overline{el} + \frac{1}{1-\kappa_E \varphi_g}(g_{1t} - \bar{g}_1) - \frac{1-\lambda_{gr}}{1-\kappa_E \varphi}(r_{ft} - \bar{r}_f). \tag{C.26}$$

Taking the exponential of this and relabelling $\overline{el}$ as the parameter υ, we get the expression (32).

D Model calibration

D.1 Macroeconomic variables

We build the short-term real interest rate from long-term yields, as in Beeler and Campbell (2012). We regress the 10-year nominal Treasury yield less the next year's inflation on the yield and on lagged inflation over the post-war period, take the fitted value as the expected riskfree rate ten years ahead, $\hat{f}_{10,t}$, and invert equation (3) to recover r_{ft}. Using long-term rates smooths short-term variation that is outside our model and extends the series back to 1791. We estimate the dynamics of growth (7) and stock returns (12) jointly by the simulated method of moments, minimizing the squared percentage errors between simulated and data moments, with real GDP per capita from the Global Macro Database and S&P 500 total returns from Shiller for 1900–2021; the growth moments and the correlations use data from 1960. Section 3.1 of the Supplementary Codebook gives the regression, the estimation procedure, and the fit to each moment.

D.2 Idiosyncratic income process

We estimate the idiosyncratic component of earnings as in Appendix B.1 of Catherine (2022), targeting the standard deviation and skewness of log earnings growth in the Social Security data of Guvenen et al. (2014), with two differences. First, we feed per capita GDP growth into the simulation before 1978 and scale mean earnings growth over 1978–2011 to the volatility of GDP growth. Second, we account for the two earners of a typical household: we estimate the process for a single earner, simulate couples by matching individuals at random, with the higher earner receiving 70% of household income, and estimate the process again on the couples' moments. Section 3.2 of the Supplementary Codebook reports both sets of estimates and the fit to each moment.

D.3 Income tax rates

Households pay income tax on labor income and Social Security benefits at the seven marginal rates of the 2020 schedule, from 10% to 37%, with the bracket limits divided by the wage index. Section 3.3 of the Supplementary Codebook lists the brackets.

D.4 Entrepreneurs

We calibrate the private business income (30) and wealth (32) processes to entrepreneurs in the SCF: households aged 18 to 60 whose main respondent actively manages a private business and whose business and farm income exceeds their wage income. The mean log valuation ratio matches the aggregate ratio of business value to business income net of the entrepreneur's predicted wage. The life-cycle profile of business income is a cubic in age, the volatilities of the permanent component come from a regression of the variance of log business income on age, and the entry probability is the change in a cubic fitted to the share of entrepreneurs at each age. Section 3.4 of the Supplementary Codebook gives the formulas and the estimates.

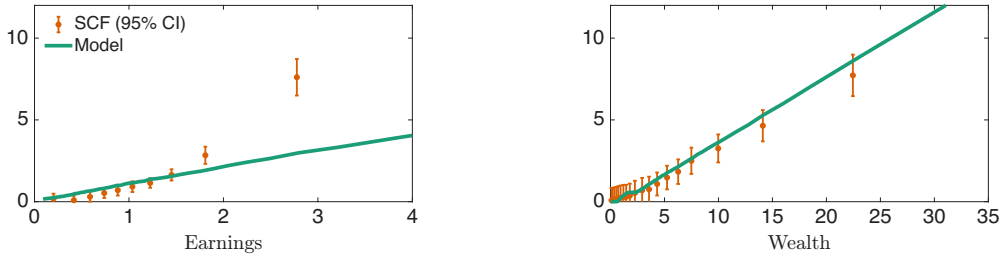
D.5 Model fit by income and wealth

Figures D.3 and D.4 report average equity and housing holdings, respectively, by income and wealth for households aged 40–45. The model reproduces the empirical pattern that richer and higher-income households allocate more to both equities and housing—not only in levels, but also as a share of wealth. A well-known challenge for life-cycle models is to rationalize the very low stock-market participation and equity shares among young and

low-income households. In our framework, this margin is primarily driven by countercyclical labor-income risk (Catherine, 2022), which raises the effective risk exposure of these households in downturns and dampens their incentives to hold equities. The fit is similar with and without entrepreneurs.

Figure D.3: Equity holdings at age 40–45

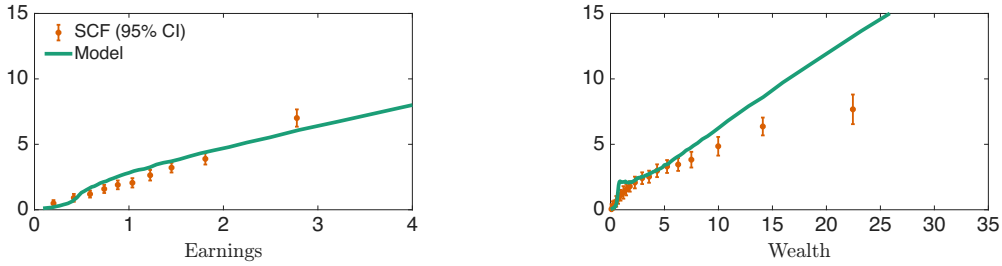
A. By income level B. By wealth level



Note: This figure reports the amount of wealth invested in stocks in the model and in the data, for households between age 40 and 45, by level of income and wealth. Equity wealth, income, and wealth are measured in units of the national wage index. Entrepreneurs are excluded.

Figure D.4: Housing wealth at age 40–45

A. By income level B. By wealth level



Note: This figure reports the amount of wealth invested in housing in the model and in the data, for households between age 40 and 45, by level of income and wealth. Housing wealth, income, and wealth are measured in units of the national wage index. Entrepreneurs are excluded.

E Numerical appendix

We use two properties of the model to solve it numerically. First, the wage index $\bar{L}_t$ and the house price P_{ht} drop out of the state space: letting $\hat{Z}_t$ denote the wage-indexed quantity $Z_t/\bar{L}_t$, there is a function $\hat{V}$ such that

$$V_{it} = \bar{L}_t^{1-\gamma} P_{ht}^{-\nu(1-\gamma)} \hat{V}(\hat{W}_{it}, r_{ft}, g_{1t}, z_{it}, a_{it}, \hat{\text{AIYE}}_{it}), \tag{E.1}$$

provided that the agent chooses $\{\hat{C}_{it}, \hat{P}_{ht}H_{it}\}$ instead of $\{C_{it}, H_{it}\}$. Second, because all three risky assets are exposed to interest-rate shocks, we restate the portfolio problem as a choice of $\{\varepsilon_{r,it}, \pi_{s,it}, \pi_{h,it}\}$, where $\varepsilon_{r,it}$ is the interest-rate sensitivity of the total portfolio. These choices map to the long-term bond share $\pi_{n,it} = (\varepsilon_{r,it} - \pi_{s,it}\lambda_{sr} - \pi_{h,it}\sigma_{n_h})/\sigma_n$, and $\varepsilon_{r,it}$, unlike $\pi_{n,it}$, does not depend on the maturity of the bond. Section 4 of the Supplementary Codebook proves the first property and derives the second.

F Simulation of the overlapping-generations model

F.1 Aggregate shocks

The simulation feeds in four aggregate series: the riskfree rate and its shocks from Appendix D.1, growth in real GDP per capita, stock returns from the S&P 500, and house prices from the Case-Shiller index. Stock returns and house capital gains enter net of the part driven by interest-rate shocks, and each household also draws an idiosyncratic house-price shock with variance $\tilde{\sigma}_h^2$. When not using data, we randomly assign agents aggregate shocks from 50 randomly generated time series; we compute the top 10% wealth share within each group of shocks and then average across the 50 groups. Section 3.5 of the Supplementary Codebook gives the details.

F.2 Inheritance and intergenerational income persistence

To model intergenerational income persistence, we assume a correlation between the levels of persistent income of parents and their children at labor market entry. Specifically, we model initial persistent income as an AR(1) process: $z_{i,t_0} = \rho_{z_0} z_{t_0-30}^{\text{parent}_i} + \sigma_{z_0}\epsilon_{z_0}$, where z_{i,t_0} denotes the child's initial value of z upon entering the labor force, and $z_{t_0-30}^{\text{parent}_i}$ is the corresponding value for the parent 30 years earlier. We calibrate the parameters ρ_{z_0} and σ_{z_0} using two moment conditions. First, the variance of z_{i,t_0} must equal σ_α^2, a parameter chosen to match the observed level of income inequality. Second, we set ρ_{z_0} to reproduce the empirical slope of the relationship between the child's and the parent's income ranks at age 30, as documented in Chetty et al. (2014). Given the rest of our labor income process, these conditions imply $\rho_{z_0} = 0.4537$ and $\sigma_{z_0} = 0.9803$. Figure F.5 plots the child's income rank against the parent's income rank, providing the model counterpart to Figure II in Chetty et al. (2014).

Figure F.5: Child income rank vs parent income rank in the model



F.3 Welfare across cohorts

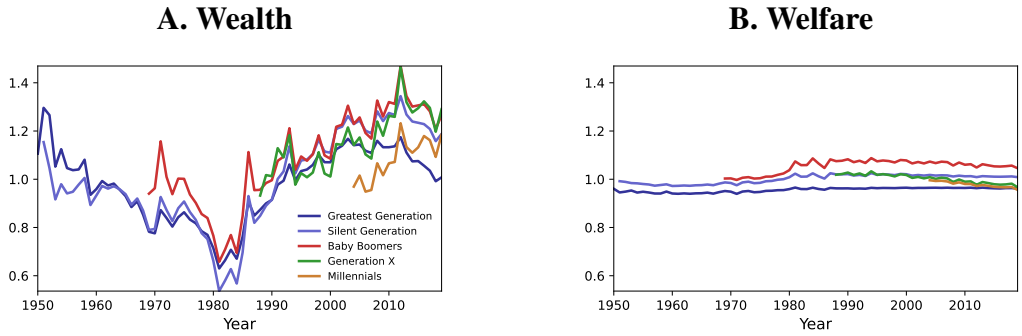
Because agents cannot hedge interest-rate shocks that occur before they are born, lifetime utility varies across cohorts even when it is equalized within them. Figure F.6 illustrates this by showing average wealth and lifetime utility for five generations. Both are measured relative to an average of simulations with a random time series of interest-rate shocks, so they should be read as percentage deviations from the historical norm. Panel A shows that the large increase in interest rates from 1960–1985 reduced wealth for every generation that lived through it. However, the generations entering the labor force closest to the high-rate period of the 1980s—the baby boomers—had the benefit of saving at substantially higher rates, leaving them with higher lifetime wealth than other cohorts.

Panel B, which instead reports lifetime utility, reveals two things. First, the cohorts that entered the labor force at a time of high interest rates, and which subsequently enjoyed capital gains, did experience above-average lifetime welfare; this is because agents cannot fully hedge interest-rate shocks across generations. Second, comparing Panel B with Panel A, fluctuations in welfare across cohorts are substantially smaller than fluctuations in wealth.

G Derivation of the linearized model

This section lays out the details of the linearization and analytical solutions presented in Section 4. The approach follows that of Campbell and Viceira (2001), except we add finite lives, intertemporal income, and multiple assets. To fully understand the economics, we first solve for policies in the general case of recursive utility (i.e., disentangling risk aversion and the EIS), then reduce to the time-additive case in the main text. Throughout

Figure F.6: Cross-generation welfare inequality



Note: This figure reports average wealth (Panel A) and lifetime utility (Panel B) for the five most recent generations in the simulation. Lifetime utility is calculated as $\bar{U}_{it} = \mathbb{E}_t \left[(1-\gamma) \sum_{s=t+a_{\min}-a}^{t+a_{\max}-a} \beta^{s-t} u(C_{is}, H_{is})\right]^{\frac{1}{1-\gamma}}$, where u is utility under optimal policies. This is a weighted average of realized consumption and expected consumption over the life cycle, without the bequest motive. After agents die in the simulation, they are assigned their last living value of U_i for the remainder of the simulation. The specification presented is the average of U_{it} with historical interest-rate shocks and growth, divided by the hypothetical U_{it} with randomly drawn interest-rate shocks. All other shocks are drawn randomly.

this section, we suppress i indices and state approximate (i.e., linearized) equalities as exact.

G.1 Linearized conditions

Suppose that there is no intertemporal income or housing, so the budget constraint (28) simplifies to[35]

$$W_{t+1} = (W_t - C_t) R_{W,t+1}. \tag{G.1}$$

The first-order condition for a recursive-utility agent takes the familiar form

$$1 = \mathbb{E}_t \left[(\beta(1-m_t))^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{-\theta/\psi} R_{W,t+1}^{\theta-1} R_{j,t+1} \right], \tag{G.2}$$

where $\beta(1-m_t)$ is mortality-adjusted patience, ψ is the EIS, $\theta = (1-\gamma)/(1-1/\psi)$, and $R_j \in \{R_f, R_n, R_W\}$. The analytical solution follows from linearizing this budget constraint and first-order condition.

Let lowercase letters denote logs and the Δ operator denote first differences. Scaling the budget constraint (G.1) by financial wealth W_t, taking logs, and linearizing $\log(1 - e^{c_t - w_t})$ around $c_t - w_t = \log(1 - \beta(1-m_t))$ implies

$$\Delta w_{t+1} = \kappa_w(m_t) + \left(1 - \frac{1}{\rho_c(m_t)}\right)(c_t - w_t) + r_{w,t+1}, \tag{G.3}$$

where $\rho_c(m_t) = \beta(1-m_t)$ and $\kappa_w(m_t) = \log \rho_c(m_t) + \frac{(1-\rho_c(m_t)) \log(1-\rho_c(m_t))}{\rho_c(m_t)}$.[36] (Notice

[35] It would be fine to assume rented housing, in which case consumption here represents total consumption of both goods and housing. We abstract from this for simplicity.

[36] In infinite-horizon models like that of Campbell and Viceira (2001), one typically chooses $\rho_c = 1 -$

that, as $m_t \to 1$, $c_t \to w_t$; agents who will die almost surely consume everything.) We can also get the linearized approximation to the log wealth return

$$r_{w,t+1} = r_{ft} + \pi_t(r_{n,t+1} - r_{ft}) + \frac{1}{2}\pi_t(1-\pi_t)\text{var}_t(r_{n,t+1}). \tag{G.4}$$

This expression is a discretization of the exact continuous-time law of motion. Finally, log-linearize the Euler equation (G.2) up to a second order:

$$\begin{aligned} 0 = \theta\log(\beta(1-m_t)) + \mathbb{E}_t\left[-\frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)r_{w,t+1} + r_{j,t+1}\right] \\ + \frac{1}{2}\text{var}_t\left(-\frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)r_{w,t+1} + r_{j,t+1}\right). \end{aligned} \tag{G.5}$$

Substituting in $r_j = r_n$ and then $r_j = r_f$ and subtracting the two equations implies the risk premium on the long-term bond

$$\mathbb{E}_t[r_{n,t+1} - r_{ft}] + \frac{1}{2}\text{var}_t(r_{n,t+1}) = \frac{\theta}{\psi}\text{cov}_t(r_{n,t+1}, \Delta c_{t+1}) + (1-\theta)\text{cov}_t(r_{n,t+1}, r_{w,t+1}). \tag{G.6}$$

Using the decomposition

$$\Delta c_{t+1} = (c_{t+1} - w_{t+1}) - (c_t - w_t) + \Delta w_{t+1} \tag{G.7}$$

and the expression for Δw_{t+1} from the linearized budget constraint (G.3), we can rewrite

$$\text{cov}_t(r_{n,t+1}, \Delta c_{t+1}) = \text{cov}_t(r_{n,t+1}, c_{t+1} - w_{t+1}) + \text{cov}_t(r_{n,t+1}, r_{w,t+1}).$$

Substituting this and the fact that

$$\text{cov}_t(r_{n,t+1}, r_{w,t+1}) = \pi_t \text{var}_t(r_{n,t+1}) \tag{G.8}$$

into (G.6) and using $\theta/\psi + 1 - \theta = \gamma$ implies the solution

$$\pi_t = \frac{1}{\gamma}\frac{\mathbb{E}_t[r_{n,t+1} - r_{ft}] + \frac{1}{2}\text{var}_t(r_{n,t+1})}{\text{var}_t(r_{n,t+1})} - \frac{1-1/\gamma}{1-\psi}\frac{\text{cov}_t(r_{n,t+1}, c_{t+1} - w_{t+1})}{\text{var}_t(r_{n,t+1})}. \tag{G.9}$$

As explained in the main text, the first term is the myopic risk-return portfolio; the second is intertemporal hedging of interest-rate risk.

Another fact that will be useful is that the condition (G.5) for wealth returns ($r_j = r_w$) simplifies to

$$\mathbb{E}_t[\Delta c_{t+1}] = \psi\log(\beta(1-m_t)) + \psi\mathbb{E}_t\left[r_{w,t+1}\right] + \frac{1}{2}\frac{\theta}{\psi}\text{var}_t\left(\Delta c_{t+1} - \psi r_{w,t+1}\right). \tag{G.10}$$

Using fact (G.8) and the decomposition of Δc from (G.7), the variance term can be rewritten as

$\exp\{\mathbb{E}[c_t - w_t]\}$, which reduces to $\rho_c = \beta$ for an EIS of 1. Here, to capture the effect of aging, we linearize instead around the unit-EIS solution, which is exact in our model.

$$\begin{aligned}\text{var}_t\left(\Delta c_{t+1} - \psi r_{w,t+1}\right) &= \text{var}_t\left(c_{t+1} - w_{t+1} + (1-\psi) r_{w,t+1}\right) \\ &= \text{var}_t\left(c_{t+1} - w_{t+1}\right) + (1-\psi)^2 \pi_t^2 \text{var}_t\left(r_{n,t+1}\right) \\ &\quad + 2(1-\psi)\pi_t \text{cov}_t\left(r_{n,t+1}, c_{t+1} - w_{t+1}\right).\end{aligned} \tag{G.11}$$

We will use these expressions to solve for the equilibrium consumption-wealth ratio.

G.2 Optimal policies in the linearized model

We will now solve for the optimal consumption and portfolio choices using the conditions derived above. Conjecture that the optimal consumption-wealth ratio takes the form

$$c_t - w_t = \log(1 - \beta(1 - m_t)) + (1-\psi)(\varrho_{0t} + \varrho_{rt} r_{ft}), \tag{G.12}$$

for some functions $\varrho_{0t} = \varrho_0(\{m_s\}_{s \geq t})$ and $\varrho_{rt} = \varrho_r(\{m_s\}_{s \geq t})$ of the future mortality probabilities. Increasing utility implies the boundary conditions $\lim_{m \to 1}(1-\psi)\varrho_0(m) = 0$ and $\lim_{m \to 1} \varrho_r(m) = 0$. This conjecture implies

$$(1-\psi)^{-1}\text{cov}_t(r_{n,t+1}, c_{t+1} - w_{t+1}) = \varrho_{r,t+1}\text{cov}_t(r_{n,t+1}, r_{f,t+1}) = -\varrho_{r,t+1}\sigma_n\sigma_r.$$

Substituting this expression into (G.9), we obtain

$$\pi_t = \frac{1}{\gamma}\frac{\mu_n + \frac{1}{2}\sigma_n^2}{\sigma_n^2} + \left(1 - \frac{1}{\gamma}\right)\varrho_{r,t+1}\frac{\sigma_r}{\sigma_n} = a_0 + a_r \varrho_{r,t+1},$$

which, combined with (10), is our expression for the optimal share in the n-period bond (35).

To solve for ϱ_0 and ϱ_r, notice that substituting this solution for π into the expectation of our log-linearized wealth return (G.4) implies

$$\begin{aligned}\mathbb{E}_t[r_{w,t+1}] &= r_{ft} + \pi_t \mu_n + \frac{1}{2}\pi_t(1-\pi_t)\sigma_n^2 \\ &= r_{ft} + \left(a_0\mu_n + \frac{1}{2}(a_0 - a_0^2)\sigma_n^2\right) + \left(a_r\mu_n + \frac{1}{2}(a_r - 2a_0a_r)\sigma_n^2\right)\varrho_{r,t+1} - \frac{1}{2}a_r^2\sigma_n^2\varrho_{r,t+1}^2 \\ &= r_{ft} + d_0 + d_1\varrho_{r,t+1} - d_2\varrho_{r,t+1}^2.\end{aligned} \tag{G.13}$$

It also implies

$$\begin{aligned}\text{var}_t\left(c_{t+1} - w_{t+1}\right) &= (1-\psi)^2\varrho_{r,t+1}^2\sigma_r^2, \\ (1-\psi)^2\pi_t^2\text{var}_t\left(r_{n,t+1}\right) &= (1-\psi)^2(a_0^2 + 2a_0a_r\varrho_{r,t+1} + a_r^2\varrho_{r,t+1}^2)\sigma_n^2, \\ (1-\psi)\pi_t\text{cov}_t\left(r_{n,t+1}, c_{t+1} - w_{t+1}\right) &= (1-\psi)^2(a_0 + a_r\varrho_{r,t+1})(-\varrho_{r,t+1}\sigma_n\sigma_r).\end{aligned}$$

Substituting these three equations into (G.11), we have

$$\text{var}_t\left(\Delta c_{t+1} - \psi r_{w,t+1}\right) = (1-\psi)^2(g_0 + g_1\varrho_{r,t+1} + g_2\varrho_{r,t+1}^2) \tag{G.14}$$

for constants g_j. Finally, substituting our log-linearized budget constraint (G.3) into the decomposition (G.7) and applying our conjecture (G.12), we have

$$\begin{aligned}\mathbb{E}_t[\Delta c_{t+1}] &= \mathbb{E}_t[c_{t+1} - w_{t+1}] - \rho_c(m_t)^{-1}(c_t - w_t) + \kappa_w(m_t) + \mathbb{E}_t[r_{w,t+1}] \\ &= (1-\psi)(\varrho_{0,t+1} + \varrho_{r,t+1}((1-\varphi)\bar{r}_f + \varphi r_{ft})) \\ &\quad - \rho_c(m_t)^{-1}(1-\psi)(\varrho_{0t} + \varrho_{rt} r_{ft}) + \log \rho_c(m_t) \\ &\quad + \log \frac{1-\rho_c(m_{t+1})}{1-\rho_c(m_t)} + \mathbb{E}_t[r_{w,t+1}].\end{aligned} \tag{G.15}$$

Substituting (G.14), (G.15), and (G.13) into the Euler equation for wealth returns (G.10), then collecting coefficients on r_{ft}, implies the difference equation $\varphi\varrho_{r,t+1} = \rho_c(m_t)^{-1}\varrho_{rt} - 1$. Now iterate forward and use the boundary condition $\lim_{t\to\infty} \varrho_{rt} = 0$:

$$\begin{aligned}\varrho_{rt} &= \rho_c(m_t)(\varphi\varrho_{r,t+1} + 1) \\ &= \rho_c(m_t) + \varphi\rho_c(m_t)\rho_c(m_{t+1}) + \varphi^2\rho_c(m_t)\rho_c(m_{t+1})\rho_c(m_{t+2}) + \dots \\ &= \beta(1-m_t)\left(1 + \sum_{j=1}^{\infty} \varphi^j \beta^j \prod_{k=1}^{j}(1-m_{t+k})\right) = \sum_{j=1}^{\infty} \varphi^{j-1}\beta^j p_{t,t+j}.\end{aligned}$$

The lower are the probabilities of future survival $p_{t,t+j}$, the less relevant are fluctuations in the interest rate to consumption and portfolio choices. For reference, note that, for infinitely lived agents ($m_t = 0$ for all t), this converges to $\varrho_r = \rho_c/(1-\varphi\rho_c)$, the result from Campbell and Viceira (2001).

Collecting the remaining constant terms implies the difference equation $\varrho_{0,t+1} = \rho_c(m_t)^{-1}\varrho_{0t} - q_{0t}$ for deterministic constant $q_{0t} \equiv \varrho_{r,t+1}(1-\varphi)\bar{r}_f + \log\rho_c(m_t) + (1-\psi)^{-1}(\log(1-\rho_c(m_{t+1})) - \log(1-\rho_c(m_t))) + d_0 + d_1\varrho_{r,t+1} - d_2\varrho_{r,t+1}^2 - \frac{1}{2}(\gamma-1)(g_0 + g_1\varrho_{r,t+1} + g_2\varrho_{r,t+1}^2)$. We can similarly iterate this expression forward with terminal condition $(1-\psi)\varrho_0 \to 0$ to arrive at $\varrho_{0t} = \rho_c(m_t)(q_{0t} + \varrho_{0,t+1}) = \sum_{j=1}^{\infty}\beta^j p_{t,t+j} q_{0,t+j-1}$. Note that $\lim_{m\to 1}\rho(m)q_{0t} = 0$, so ϱ_{0t} is finite and converges to zero as $m \to 1$. This verifies the conjecture.

G.3 Adding labor income and Social Security

We now introduce a deterministic stream of labor income L with Social Security taxes T and benefits B. The present values of labor income (human capital) H and Social Security S are as stated in the main text.

As we did with the wealth return above, let us linearize the returns on human capital and Social Security wealth using a continuous-time approximation. For human capital, the

log return is

$$r_{H,t+1} = r_{ft} + \mu_{Ht} + \underbrace{\left(\sum_{j=1}^{t_{\text{ret}}-t} \omega_{jt}^{H} \left(\frac{\sigma_j}{\sigma_n} \right) \right)}_{\pi_t^H} (r_{n,t+1} - r_{ft})$$

where

$$\omega_{jt}^{H} = \frac{p_{t,t+j} P_{jt} L_{t+j}}{\sum_{j'=1}^{t_{\text{ret}}-t} p_{t,t+j'} P_{j't} L_{t+j'}} = \frac{p_{t,t+j} P_{jt} L_{t+j}}{H_t}$$

is the value weight of the jth labor-payment, and therefore π^H is a value-weighted rate-sensitivity adjustment. More specifically, π^H represents the percent holdings of n-period bonds implicit in the human capital asset. To see this, note that the rate sensitivity implied by π^H is

$$\pi_t^H \frac{\sigma_n}{\sigma_r} = \sum_{j=1}^{t_{\text{ret}}-t} \omega_{jt}^{H} \left(\frac{\sigma_j}{\sigma_r} \right) = \sum_{j=1}^{t_{\text{ret}}-t} \omega_{jt}^{H} \varepsilon_r(P_{j,t+1}) = \varepsilon_r(H_{t+1}).$$

In words, a portfolio with π^H percent allocated to the n-period bond has the exact same interest-rate sensitivity as the human capital asset. Identical logic leads us to the log return on Social Security,

$$r_{S,t+1} = r_{ft} + \mu_{St} + \underbrace{\left(\sum_{j=1}^{\infty} \omega_{jt}^{S} \left(\frac{\sigma_j}{\sigma_n} \right) \right)}_{\pi_t^S} (r_{n,t+1} - r_{ft}),$$

$$\omega_{jt}^{S} = \omega_{jt}^{B} - \omega_{jt}^{T} = \frac{p_{t,t+j} P_{jt} (B_{t+j} - T_{t+j})}{S_t}.$$

Now, as in the main text, define total wealth as

$$\overline{W}_t = W_t + (L_t + H_t) + (B_t - T_t + S_t). \tag{G.16}$$

(Recall that H and S do not include their contemporaneous "dividends," so we must add them back in this expression.) Grossing up at the rates of return on these assets implies

$$\overline{W}_{t+1} = (W_t + L_t + B_t - T_t - C_t) R_{W,t+1} + H_t R_{H,t+1} + S_t R_{S,t+1}. \tag{G.17}$$

Multiplying and dividing by $\overline{W}_t - C_t$, we have the dynamic budget constraint

$$\overline{W}_{t+1} = (\overline{W}_t - C_t) R_{\overline{W},t+1},$$

where the return on total wealth is

$$
\begin{aligned}
R_{\overline{W},t+1} &= \left(\frac{W_t + L_t + B_t - T_t - C_t}{\overline{W}_t - C_t}\right) R_{W,t+1} + \left(\frac{H_t}{\overline{W}_t - C_t}\right) R_{H,t+1} \\
&\quad + \left(\frac{S_t}{\overline{W}_t - C_t}\right) R_{S,t+1} \\
&= \alpha_{Wt} R_{W,t+1} + \alpha_{Ht} R_{H,t+1} + \alpha_{St} R_{S,t+1},
\end{aligned}
$$

and the return on marketable wealth R_W is as it was in the original problem.

Using the same linearization technique as before, the log total-wealth return can be approximated as

$$
r_{\overline{w},t+1} = r_{ft} + \bar{\mu}_t + \bar{\pi}_t(r_{n,t+1} - r_{ft}) + \frac{1}{2}\bar{\pi}_t(1 - \bar{\pi}_t)\sigma_n^2,
$$

where $\bar{\mu}_t = \alpha_{Ht}\mu_{Ht} + \alpha_{St}\mu_{St}$ is a value-weighted drift term from the intertemporal endowments, and

$$
\bar{\pi}_t = \alpha_{Wt}\pi_t + \alpha_{Ht}\pi_t^H + \alpha_{St}\pi_t^S \tag{G.18}
$$

is the value-weighted average of positions in the long-term bond—that is, the percentage of total wealth invested in the bond. Other than the presence of $\bar{\mu}$, this budget constraint is identical in form to that of the problem with no labor income or Social Security. Following the same steps from before, we conclude that $\bar{\pi}_t = \pi_t^*$, where π_t^* is the optimal solution without intertemporal income. Substituting this into (G.18) and solving for π_t, we see that the optimal allocation to the asset from financial wealth is

$$
\pi_t = \pi_t^* + \left(\frac{H_t}{W_t + L_t + B_t - T_t - C_t}\right)(\pi_t^* - \pi_t^H) + \left(\frac{S_t}{W_t + L_t + B_t - T_t - C_t}\right)(\pi_t^* - \pi_t^S).
$$

In the main text, we slightly simplify notation by redefining wealth W_t to include the contemporaneous income and consumption flows (thus far, we have assumed that it *excludes* these components). Doing this gives us the final expression (37).

G.4 Solution to the value function

This section solves for the value function under optimal policies in closed form. Under recursive preferences, the transformed value function (44) is implicitly defined by the aggregator

$$
U_t = \left[(1 - \beta(1 - m_t))C_t^{1-1/\psi} + \beta(1 - m_t)\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}.
$$

By Euler's Theorem,

$$
U_t = \frac{\partial U_t}{\partial C_t} C_t + \mathbb{E}_t\left[\frac{\partial U_t}{\partial U_{t+1}} U_{t+1}\right],
$$

where

$$\frac{\partial U_t}{\partial C_t} = (1-\beta(1-m_t))\left(\frac{U_t}{C_t}\right)^{1/\psi} \quad \text{and} \quad \frac{\partial U_t}{\partial U_{t+1}} = \beta(1-m_t)U_t^{1/\psi}\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{\gamma-1/\psi}{1-\gamma}} U_{t+1}^{-\gamma}.$$

Noting that the stochastic discount factor

$$M_{t+1} = \beta(1-m_t)\left(\frac{C_{t+1}}{C_t}\right)^{-1/\psi}\left[\frac{U_{t+1}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]^{1/(1-\gamma)}}\right]^{-(\gamma-1/\psi)},$$

and that

$$\frac{(\partial U_t/\partial U_{t+1})(\partial U_{t+1}/\partial C_{t+1})}{\partial U_t/\partial C_t} = M_{t+1},$$

we have that

$$\frac{U_t}{\partial U_t/\partial C_t} = C_t + \mathbb{E}_t\left[M_{t+1}\frac{U_{t+1}}{\partial U_{t+1}/\partial C_{t+1}}\right] = \sum_{j=0}^{\infty}\mathbb{E}_t\left[M_{t,t+j}C_{t+j}\right] = \overline{W}_t,$$

since total wealth is the present value of consumption. Substituting the expression for $\partial U_t/\partial C_t$ and noting that consumption is at an optimum ($C_t = C_t^*$), we get the solution (45).

G.5 Adding multiple assets

Consider the setting with J non-redundant risky assets in Section 4.3, summarized by (39) and (40). In this case, the linearized wealth return becomes

$$r_{w,t+1} = r_{ft} + \pi_t^\top(r_{t+1} - r_{ft}\iota) + \frac{1}{2}\left(\pi_t^\top \text{diag}(\Sigma^\top\Sigma) - \pi_t^\top\Sigma^\top\Sigma\pi_t\right).$$

The log-linearized Euler equations are the same for each asset j, so by the exact same logic, we have the vector of optimal portfolio weights

$$\begin{aligned}\pi_t^* = \frac{1}{\gamma}(\Sigma^\top\Sigma)^{-1}\left(\mathbb{E}_t[r_{t+1} - r_{ft}\iota] + \frac{1}{2}\text{diag}(\Sigma^\top\Sigma)\right)\\ - \left(1-\frac{1}{\gamma}\right)(\Sigma^\top\Sigma)^{-1}\frac{\text{cov}_t(r_{t+1}, c_{t+1}-w_{t+1})}{1-\psi}.\end{aligned}$$

Also by the same logic, the consumption-wealth ratio $c_t - w_t$ continues to take the same form, namely affine in r_{ft}. Consequently, substituting in the expected log returns and the fact that

$$\text{cov}_t(r_{t+1}, r_{f,t+1}) = \Sigma^\top\sigma_r,$$

we get the optimal portfolio rule π_t^* and the resulting rate sensitivity $\varepsilon_r^*(W_{t+1})$ stated in the main text.

G.6 Optimal consumption plan in the limit

This section derives the optimal consumption-investment strategy in the limit as risk aversion γ approaches infinity and the EIS ψ approaches zero. We first show that, in the limit as

$\gamma \to \infty$ and $\psi \to 0$, the optimal consumption plan is constant and financed by a perpetuity. We then prove that both of these conditions (i.e., both $\gamma \to \infty$ and $\psi \to 0$, not just one or the other) are necessary for this to be true.[37]

First, let us consider what happens when $\gamma = 1/\psi \to \infty$. The first-order condition when $\gamma = 1/\psi$ is $1 = \mathbb{E}_t\left[\beta(1-m_t)\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{j,t+1}\right]$. Conjecture that the optimal consumption policy is a deterministic constant $C_t = \bar{C}_t$. Substituting this conjecture into the first-order condition implies the recursion $\bar{C}_t = \left(\beta(1-m_t)\mathbb{E}_t[R_{j,t+1}]\right)^{-1/\gamma}\bar{C}_{t+1}$. Now taking the limit as $\gamma \to \infty$ implies that $\bar{C}_t = \bar{C}_{t+1} = \bar{C}$, meaning that consumption is indeed deterministic and in fact time-invariant.

Next, we show that a constant consumption plan is financed by the optimal portfolio strategy π_t^*, and that $\gamma = 1/\psi \to \infty$ is necessary, not just sufficient.[38] In order for the consumption plan to be constant, we need two conditions to be satisfied. First, expected consumption growth must be invariant to the level of interest rates; and second, unexpected consumption growth must be invariant to shocks to interest rates. That is, we require

$$\frac{d\mathbb{E}_t[\Delta c_{t+1}]}{dr_{ft}} = 0 \quad \text{and} \quad \frac{d(\Delta c_{t+1})}{d\epsilon_{r,t+1}} = 0. \tag{G.19}$$

Using the optimal policy for $c_t - w_t$, consumption growth is

$$\Delta c_{t+1} = \Delta w_{t+1} + (1-\psi)(\varrho_{0,t+1} - \varrho_{0t}) + (1-\psi)(\varrho_{r,t+1} r_{f,t+1} - \varrho_{rt} r_{ft}), \tag{G.20}$$

with $\Delta w_{t+1} = \kappa_w(m_t) + \left(1 - \frac{1}{\rho_c(m_t)}\right)(c_t - w_t) + r_{w,t+1}$. The unexpected component of this is

$$\Delta c_{t+1} - \mathbb{E}_t[\Delta c_{t+1}] = -\pi_t \sigma_n \epsilon_{r,t+1} + (1-\psi)\varrho_{r,t+1}\sigma_r \epsilon_{r,t+1}. \tag{G.21}$$

Substituting in the optimal portfolio and differentiating, this implies

$$\frac{d(\Delta c_{t+1})}{d\epsilon_{r,t+1}} = -\frac{\mu_n + \sigma_n^2/2}{\gamma\sigma_n} + \left(\frac{1}{\gamma} - \psi\right)\varrho_{r,t+1}\sigma_r. \tag{G.22}$$

For expected consumption growth, differentiate (G.15) with respect to r_{ft} to see that $\frac{d\mathbb{E}_t[\Delta c_{t+1}]}{dr_{ft}} = (1-\psi)(\varphi\varrho_{r,t+1} - \rho_c(m_t)^{-1}\varrho_{rt}) + 1$. Recalling that $\varphi\varrho_{r,t+1} = \rho_c(m_t)^{-1}\varrho_{rt} - 1$, this simplifies to

$$\frac{d\mathbb{E}_t[\Delta c_{t+1}]}{dr_{ft}} = \psi. \tag{G.23}$$

The results then follow from (G.22) and (G.23). The consumption plan is invariant to

[37] This result was first shown by Campbell and Viceira (2001) for an infinitely lived agent with no background assets. An alternative (and closely related) proof can be found in their appendix.

[38] Because optimal policies $\{C_t^*, \pi_t^*\}$ were derived under linearized first-order conditions, this result will only be exactly true under those conditions.

interest rates if and only if $\psi = 0$ and $\gamma = 1/\psi \to \infty$. Under these preferences, the trading strategy implemented by the portfolio rule π_t^* finances a constant consumption plan (and hence replicates a buy-and-hold perpetuity). Moreover, the only way for a constant consumption plan to be consistent with the optimal policies is to have both $\gamma \to \infty$ and $\psi \to 0$ simultaneously. They are therefore both necessary and sufficient.

H Rate exposure under the observed portfolio policies

This appendix seeks to understand how deviations of households in the data from the theoretical benchmark would influence the welfare result from Figure 10. To do this, we build adjusted policy functions that reproduce the empirical interest-rate sensitivity estimates of wealth in the SCF by age, earnings, and wealth. We then simulate households under these policy functions and compare the resulting interest-rate sensitivity of lifetime consumption under the two policy functions.

Let x_{it}^* be the rate exposure of household i's portfolio at age a_{it} under the theoretical benchmark measured in interest-rate sensitivity. The "empirical policy function" is then $x_{it}^E = x_{it}^* + e_{it}$, where e_{it} is the residual. To exactly match the moments reported in Section 1.3, we proxy the deviations as

$$\hat{e}_{it} = \ell_b(a_{it}) + \kappa^y_{b,g(y_{it})} + \kappa^w_{b,g(W_{it})} \tag{H.1}$$

where $\ell_b(a)$ is the deviation in interest-rate sensitivity in six-year age bands b centered at $26, 32, \ldots, 86$, and $\kappa^y_{b,g}$ and $\kappa^w_{b,g}$ are constants for earnings group $g(y)$ and wealth group $g(W)$.[39] Since earnings stop in retirement, $\kappa^y_{b,g}$ is equal to 0 in retirement. Since changes in interest-rate sensitivity change wealth, to do the estimation we set quantile cutoffs to be what they are under the optimal policy function.

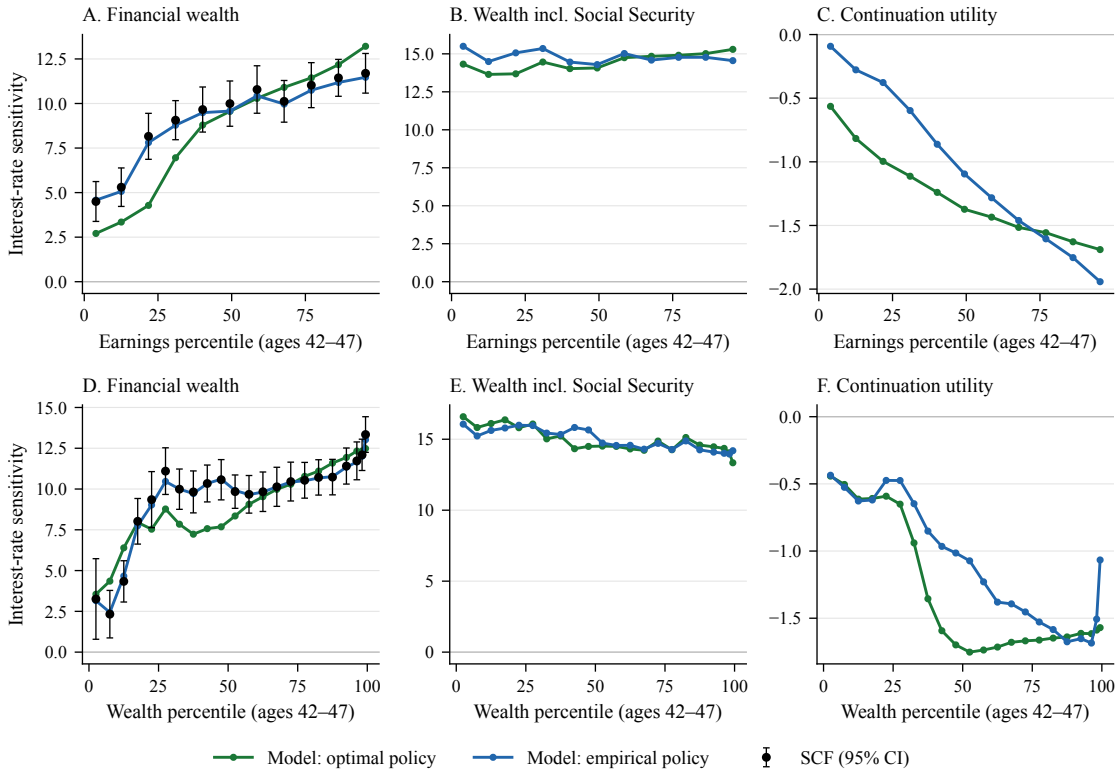
We fit the bands in order of age, since earlier bands shape the wealth that later ones inherit. Each pass simulates 20,000 households and updates every coefficient of a band at once by a Gauss–Newton step, until all of the band's targets lie within 0.15 of the interest-rate sensitivities in the SCF.

For each policy, we report three interest-rate sensitivities: marketable wealth, wealth inclusive of Social Security, and lifetime consumption. Figure H.7 plots these for households aged 42–47. The "optimal policy" lines use the unadjusted policies (as in the main

[39] Note that we match the earnings and wealth facts for ages 42–47 across all age bands, not just in the one band that we report.

text) and the "empirical policy" lines use the adjusted policies. By construction, empirical policies line up closely along the wealth and earnings gradients in the data (Panels A and D). Once Social Security is included in wealth, the differences between optimal and empirical policies become much smaller (Panels B and E).

Figure H.7: Rate sensitivity under the optimal and empirical policies, ages 42–47



Note: This figure reports the interest-rate sensitivity of financial wealth, of wealth including Social Security, and of utility for households aged 42–47 in the model, by earnings group (Panels A–C) and wealth group (Panels D–F). Green lines follow the optimal policy; blue lines follow the empirical policy (H.1) fitted to the SCF. Black dots are SCF estimates with 95% confidence intervals. Entrepreneurs are excluded.

Panels C and F report the welfare effects, as measured by changes in lifetime utility U. Overall, results are quantitatively similar under optimal and empirical policies, largely due to the importance of Social Security. If anything, under empirical policy functions, the top welfare share computed in Figure 10 would show a slight *decline* over the last four decades, because in the data the poor are slightly over-exposed—and the rich under-exposed—to fluctuations in interest rates relative to what the theoretical benchmark would suggest. This means that, for large rate declines like those we saw from 1980–2020, adjusted lifetime consumption falls slightly more for the wealthy than for the poor.