Internet Appendix for "Sovereign Default and the Decline in Interest Rates"

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A Data description

We use various series to illustrate the secular decline in interest rates in the short- and long-run. To obtain interest rates from 1311–2018, we rely on data from Schmelzing (2020). The data set contains nominal interest rate and inflation time series for several developed economies over the last eight centuries. Specifically, the data include long-term sovereign borrowing rates with an average maturity that hovers around 10 years; however, this varies over time and across countries. From these data, we plot the nominal sovereign borrowing yields for the United Kingdom, Holland, Germany, Italy, and the United States in Panel A of Figure 1. The data are collected from a variety of sources, outlined in detail in the paper and online appendix. The U.K. borrowing rates come from the Calendar of State Papers and the Bank of England. Data before 1694 for the U.K. (before the founding of the Bank of England) are not used, since the data are incomplete. Data for the Netherlands come from Dormans (1991), Weeveringh (1852), the European Central Bank, and various sources from Leiden, Haarlem, Utrecht, Schiedam, and Amsterdam. German data come from various sources from several German principalities. U.S. data come from Durand and Winn (1947), Homer and Sylla (2005), the NBER Macrohistory database, and Federal Reserve Economic Data (FRED) from the Federal Reserve Bank of St. Louis.

We also report the Bank of England (BoE) short-term lending rate (series IDGBRD) from Global Financial Data. From 1694 to 1971, the "bank rate" is used; from 1972 to 1981, the minimum lending rate is used; from 1981 to 1997, the BoE base rate is used; and from 1997 to the present, the BoE Operational interest rate is used. For more information see the Bank of England research datasets webpage.

For the U.S., our primary interest rate series—in modern times—is the effective Federal Funds Rate (series FEDFUNDS), the rate corresponding to the median volume of overnight unsecured loans between depository institutions. These data come from FRED. This series is plotted in Panel A of Figures 2 and 8. This is also the interest rate series used in all of our calibration exercises. Figure 9 presents data on the 5-year and 10-year nominal and inflation-protected Treasury bonds (FRED series DGS5, DGS10, DFII5, and DFII10). Data for interest rates in Internet Appendix E come from Jordà et al. (2019).

U.K. inflation-linked and nominal Gilts yields are taken from Global Financial Data, which sources the yields from the Bank of England. Inflation in the U.K. at one- and 5-year horizons is calculated from the monthly CPI on all items, as reported by the OECD (see Global Financial Data series IGGBR1D, IGGBR5D, and CPGBRCM).

Data on U.S. inflation expectations for Figure 6 come from the Survey of Professional Forecasters and FRED. From the Survey of Professional Forecasters, we use the 1-year, 5-year, and 10-year ahead inflation expectations in different parts of the paper. For 1-year inflation expectations, we use the median forecast of the price index for GDP (series: PGDP) as used in Angeletos et al. (2021). This is subtracted from the Fed Funds Rate to form our inflation-adjusted interest rate measure in all calibration exercises. This is also what we use to calibrate q in the model presented in Section 8. These data are also used to construct the deviation of expected inflation from realized inflation shown in Figure 7. In Figure 9, we use 5-year and 10-year expected CPI inflation to construct the difference between U.S. nominal bonds and TIPS. We do this because long-term forecasts for the price index of GDP are not available in the SPF. From FRED, we use the inflation expectations from the Surveys of Consumers of University of Michigan (series MICH), which covers short-term inflation expectations, and the expected 10-year-ahead inflation implied from Treasury Inflation-Indexed Constant Maturity Securities (series T10YIE).

Data on realized inflation in the U.S. is used in Section 2.7 where we use the annual realized change in the price level of GDP. This is also used in Figure 6 meaning we use the same forecast errors as in Angeletos et al. (2021). In Figure 5 we use the realized change in the CPI for all urban consumers.

Data on long-term earnings growth expectations come from two sources: Bordalo et al. (2024) and I/B/E/S. In particular, we use LTG which represents cumulative, annualized earnings growth over the next 3-5 years (Bordalo et al. 2024). Since Bordalo et al. (2024) present these data until the end of 2020, we fill in the remaining years with the estimate of LTG from I/B/E/SGlobal Aggregates for the S&P 500 Index. These data are used to calibrate the consumption-dividend ratio z in the generalized endowment model. Also used are the Personal Consumption Expenditures series (FRED: PCE) from FRED economic data and earnings from Robert Shiller's webpage (Shiller 2000).

Growth data come from different sources. In Tables 1–2, the U.S. growth parameter μ is set to match per capita consumption growth, series A794RX0Q048SBEA from FRED Economic Data hosted by the St. Louis Federal Reserve. In Figure 2 and Table 4, we use real per capita GDP growth rates from FRED (series A939RX0Q048SBEA) as the growth rate for the U.S. Average annual growth rates are used, which are computed using December-to-December values. When calibrating to the international evidence in Internet Appendix E, we use the real GDP growth series from Jordà et al. (2019).

Data on investment and capital stock come from the Bureau of Economic Analysis (BEA) Fixed Assets Accounts Tables. Investment data come from Table 1.5, Line 2 and capital stock data come from Table 1.1, Line 2. In these data, investment as a fraction of capital averaged 7.7% from 1984–2000 and 6.9% from 2001–2021.

Price-dividend ratio data for the U.S. from 1984 to 2021 are from the Center for Research in Security Prices (CRSP). Specifically, we use cum-dividend returns (series VWRETD) and ex-dividend returns (series VWRETX). To calculate the price-dividend ratio, we back out prices and dividends from cum- and ex-dividend returns. This series is plotted in panel B of Figure 2. We use this procedure to calculate our price-dividend ratio moments for all of the calibration exercises.

We combine price dividend ratio data with data on the cyclically adjusted price-earnings ratio (CAPE)—the price divided by the average inflation-adjusted earnings from the previous 10 years—to calibrate the model in Section 2.7. See Shiller (2000) and online data description. In particular, we use the difference between CAPE and the price dividend ratio to determine how earnings are paid out as dividends. Valuation ratio data for the international evidence in Internet Appendix E come from Jordà et al. (2019).

Finally, we obtain the Volatility Index (VIX) series from the Chicago Board Options Exchange (CBOE). The CBOE calculates the risk-neutral expected 30-day quadratic variation using option prices. There are small differences in the calculation methodology over the years; see CBOE white paper.

B Structural break test

Throughout the main text, we calibrate the model to data from two subsamples. We determine the most likely date for a structural break in the average 1-year Treasury yield.

Specifically, for each potential break year t_{break} since 1985, we estimate the regression[1]

$$y^{\$}_{b,t} - \Delta\pi_t = \beta_0 + \beta_1 \mathbb{1}\{t > t_{\text{break}}\} + \epsilon_t, \tag{B.1}$$

where $\Delta\pi_t \equiv \log(\Pi_{t+1}/\Pi_t)$ is log realized inflation.[2] Figure B.1 plots the F-statistic from this regression as a function of the break point. Evidently, 2001 stands out as the best fit for a structural break in inflation-adjusted yields.

As we mention in the main text, our choice of 2001 as a break date is also consistent with prior work studying secular changes in macroeconomic time series since the 1980s. First, Farhi and Gourio (2018) calibrate their model to two separate data samples around this date. Second, using a regression-based break test very similar to ours, Campbell et al. (2020) identify 2001 as the most likely year for a structural break in the relation between GDP growth and inflation. They find that the two series were negatively correlated prior to 2001 and became positively correlated thereafter.

C Endowment model with general EIS and constant within-period growth rates

C.1 Price-consumption ratio

Given the SDF (3), the Euler equation with respect to the consumption claim is

$$1 = \mathbb{E}_t\left[\beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R^{\theta}_{W,t+1}\right]. \tag{C.1}$$

[1] Recall that $y^{\$}_{b,t}$ is the yield on the 1-year Treasury bill and $\Delta\pi_t$ is 1-year log realized inflation, so this difference represents the ex post return on the 1-year nominal bond, (15).

[2] Results are similar using forecasts instead of realizations.

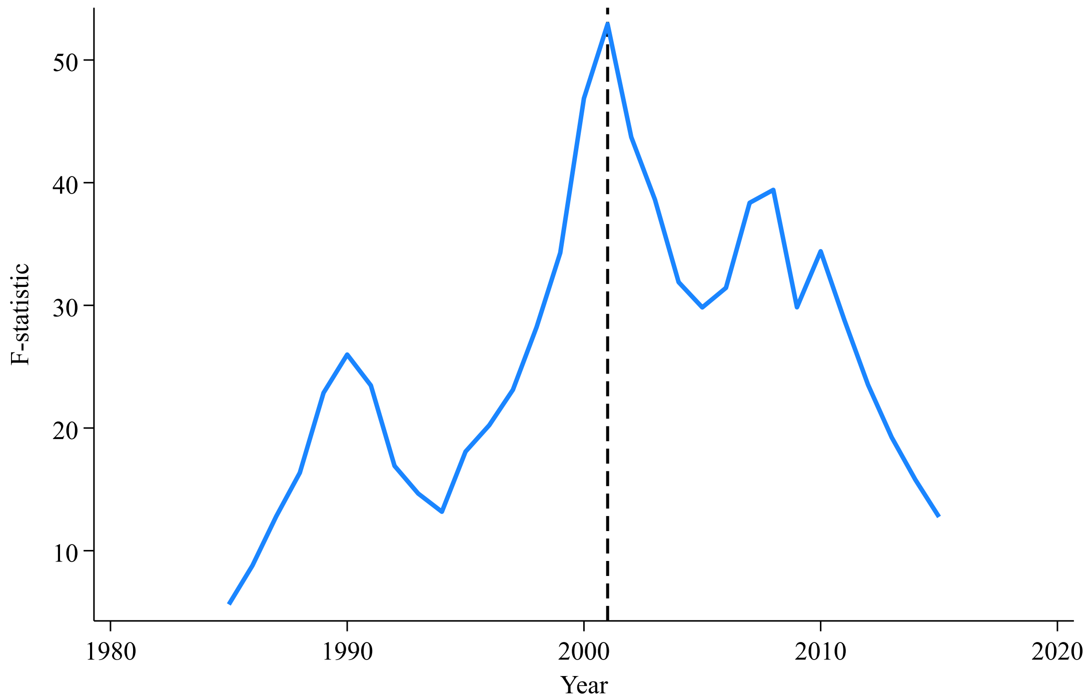

Figure B.1: Structural break test on interest rates

Notes: The figure presents F-statistics for the linear regression (B.1), estimated using OLS for all potential break dates from 1985–2015 using data on 1-year nominal Treasury bill yields less inflation.

Alt text: Line graph showing the F-statistic from a structural break test, peaking around 2001.

Conjecture a constant price-consumption ratio

$$\kappa \equiv (W_t - C_t)/C_t. \tag{C.2}$$

Substituting (C.2) into (C.1) and using $R_{W,t+1} = W_{t+1}/(W_t - C_t)$ implies

$$1 = \beta^\theta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{\theta(1-\frac{1}{\psi})} \left(\frac{\kappa + 1}{\kappa} \right)^\theta \right]. \tag{C.3}$$

Given (1–2),

$$\frac{\kappa}{\kappa+1} = \beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p((1-\eta)^{1-\gamma} - 1)\right]^{\frac{1}{\theta}}. \tag{C.4}$$

A solution exists provided that the right-hand side of (C.4) is less than one. We restrict attention to parameter combinations satisfying this restriction. Finally,

$$\kappa = \frac{\beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p((1-\eta)^{1-\gamma} - 1)\right]^{\frac{1}{\theta}}}{1 - \beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p((1-\eta)^{1-\gamma} - 1)\right]^{\frac{1}{\theta}}}, \tag{C.5}$$

verifying the conjecture.

C.2 Risk-free rate

The risk-free rate is given by the Euler equation for the risk-free asset

$$R_f = \mathbb{E}_t\left[\beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1}\right]^{-1}. \tag{C.6}$$

This simplifies to

$$R_f = \mathbb{E}_t\left[\beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{\kappa}{\kappa+1}\right)^{1-\theta}\right]^{-1}. \tag{C.7}$$

where $\kappa/(\kappa+1)$ is given by (C.4). Solving this yields the expression for the gross risk-free rate

$$R_f = \beta^{-1} e^{\frac{1}{\psi}\mu}\left[1 + p((1-\eta)^{-\gamma} - 1)\right]^{-1}\left[1 + p((1-\eta)^{1-\gamma} - 1)\right]^{\frac{\theta-1}{\theta}} \tag{C.8}$$

which implies that the log risk-free rate is given by

$$\log R_f = -\log\beta + \frac{1}{\psi}\mu - \log(1 + p((1-\eta)^{-\gamma} - 1)) + \left(\frac{\theta-1}{\theta}\right)\log(1 + p((1-\eta)^{1-\gamma} - 1)). \quad \text{(C.9)}$$

C.3 Yield and expected return with sovereign default risk

Consider the defaultable short-term government bond paying $(1 - L_{t+1})$ dollars—that is, 1 dollar in the case of no default and $1 - \lambda\eta$ dollars in the case of default. The price of this claim is obtained by solving the Euler equation

$$Q_t = \mathbb{E}_t\left[\beta^\theta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1}(1 - L_{t+1})\right], \quad \text{(C.10)}$$

which simplifies to

$$Q_t = \mathbb{E}_t\left[\beta^\theta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{\kappa}{\kappa+1}\right)^{1-\theta}(1 - L_{t+1})\right], \quad \text{(C.11)}$$

where $\kappa/(\kappa+1)$ is given by (C.4). This gives the price of the defaultable claim as

$$Q_t = \beta e^{-\frac{1}{\psi}\mu}\left[1 + p((1-\eta)^{1-\gamma} - 1)\right]^{\frac{1-\theta}{\theta}}\left[1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right]. \quad \text{(C.12)}$$

The yield on the defaultable claim is defined as $y_{b,t} \equiv -\log Q_t$, and is thus equal to the constant

$$y_b = \log R_f + \log\left(1 + p((1-\eta)^{-\gamma} - 1)\right) - \log\left(1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right), \quad \text{(C.13)}$$

where $\log R_f$ is given by (C.9). The expected excess return on the bond is the expected payoff divided by the price, less the log risk-free rate, and therefore equals

$$\log \mathbb{E}_t \left[R_{b,t+1}\right] - \log R_f = \log\left(1 + p((1-\lambda\eta) - 1)\right) + \log\left(1 + p((1-\eta)^{-\gamma} - 1)\right) - \log\left(1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right). \quad \text{(C.14)}$$

Suppose instead of being subject to outright default, the bond is a nominally risk-free asset and so the government partially defaults through inflation. Assume inflation is given by the process (9). The price of this defaultable claim is obtained by solving the Euler equation

$$Q_t^{\$} = \mathbb{E}_t\left[\beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1}\frac{\Pi_t}{\Pi_{t+1}}\right], \quad \text{(C.15)}$$

which simplifies to $Q_t^{\$} = Q_t e^{-q_t + \sigma_\pi^2/2}$ for the price Q_t given by (C.12). Subsequent results in the main text then follow straightforwardly.

D Generalized endowment model with unit EIS

For ease of exposition in what follows, we use the following notation. Standard deviations σ_x should be understood to equal $\sqrt{\sigma_x^\top \sigma_x}$ for variables x. Covariances σ_{xy} should be understood to equal $\sigma_x^\top \sigma_y$.

D.1 Discount factor and risk-free rate

With a unit EIS, the consumption-wealth ratio equals $C_t/W_t = 1-\beta$, and thus the return on wealth equals[3]

$$R_{W,t+1} = \frac{W_{t+1}}{W_t - C_t} = \beta^{-1}\frac{C_{t+1}}{C_t}. \quad \text{(D.1)}$$

[3]We re-derive this result and the form of the SDF in the production economy. See Internet Appendix G below for the derivation.

The SDF is given by the latter case in (3), namely

$$M_{t+1} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right]^{-1} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} = \mathbb{E}_t[R_{W,t+1}^{1-\gamma}]^{-1} R_{W,t+1}^{-\gamma}. \tag{D.2}$$

Substituting in the wealth return, the expectation term in the SDF equals

$$\mathbb{E}_t[R_{W,t+1}^{1-\gamma}]^{-1} = \beta^{1-\gamma} e^{(\gamma-1)\mu-(1-\gamma)^2|\sigma_C|^2/2}(1+p((1-\eta)^{1-\gamma}-1))^{-1}, \tag{D.3}$$

and so the SDF can be written

$$M_{t+1} = \beta e^{-\mu-(1-\gamma)^2|\sigma_C|^2/2-\gamma\sigma_C^\top \varepsilon_{t+1}} \frac{(1-\chi_{t+1})^{-\gamma}}{1+p((1-\eta)^{1-\gamma}-1)}, \tag{D.4}$$

and implies the risk-free rate

$$\begin{aligned} \log R_{ft} = -\log \mathbb{E}_t[M_{t+1}] &= -\log\beta + \mu + \frac{1}{2}[(1-\gamma)^2 - \gamma^2]|\sigma_C|^2 \\ &\quad - \log(1+p((1-\eta)^{-\gamma}-1)) + \log(1+p((1-\eta)^{1-\gamma}-1)). \end{aligned} \tag{D.5}$$

The term structure of inflation-protected long-term bonds is flat (that is, all yields equal the risk-free rate).

D.2 Stock prices

Given that the log consumption-dividend ratio evolves as

$$z_{t+1} = (1-\rho_z)\bar{z} + \rho_z z_t + \sigma_z^\top \varepsilon_{t+1} - \xi \log(1-\chi_{t+1}).$$

dividend growth can be expressed as

$$\frac{D_{t+1}}{D_t} = \frac{C_{t+1}}{C_t} e^{z_t - z_{t+1}} = e^{\mu + (1-\rho_z)(z_t - \bar{z}) + (\sigma_C - \sigma_z)^\top \varepsilon_{t+1}} (1 - \chi_{t+1})^{1+\xi}. \tag{D.6}$$

Henceforth, let $\mu_{Dt} = \mu + (1 - \rho_z)(z_t - \bar{z})$ and $\sigma_D = \sigma_C - \sigma_z$. For $\sigma_{Cz} \leq 0$, dividends are more volatile than consumption and rise (fall) more when consumption rises (falls). Dividends and consumption are cointegrated, so expected dividend growth is higher when the current consumption-dividend ratio z_t is larger. If we set $\sigma_z^2 = \xi = 0$ and $z_0 = \bar{z} = 1$, then the dividend is just consumption and this becomes the consumption claim.

Consider the claim to the single dividend n periods from now, D_{t+n}, and let P_{Dnt} denote its price. The price-dividend ratio on this claim will be a function of maturity n and the consumption-dividend ratio z_t:

$$\kappa_D(n, z_t) = \frac{P_{Dnt}}{D_t}. \tag{D.7}$$

The price therefore satisfies

$$\kappa_D(n, z_t) = \mathbb{E}_t \left[M_{t+1} \frac{D_{t+1}}{D_t} \kappa_D(n-1, z_{t+1}) \right]. \tag{D.8}$$

Conjecture that

$$\kappa_D(n, z_t) = \exp \{ a_D(n) + b_D(n)(z_t - \bar{z}) \} \tag{D.9}$$

for some functions $a_D(n)$ and $b_D(n)$ with $a_D(0) = b_D(0) = 0$. Noting that

$$M_{t+1} \frac{D_{t+1}}{D_t} = \beta e^{(1-\rho_z)(z_t - \bar{z}) - (1-\gamma)^2 |\sigma_C|^2/2 + (\sigma_D - \gamma\sigma_C)^\top \varepsilon_{t+1}} \frac{(1 - \chi_{t+1})^{1+\xi-\gamma}}{1 + p((1-\eta)^{1-\gamma} - 1)} \tag{D.10}$$

and

$$\kappa_D(n-1, z_{t+1}) = \exp \left\{ a_D(n-1) + b_D(n-1)(\rho_z(z_t - \bar{z}) + \sigma_z^\top \varepsilon_{t+1}) \right\} (1 - \chi_{t+1})^{-\xi b_D(n-1)}, \tag{D.11}$$

this means that

$$e^{a_D(n)+b_D(n)(z_t-\bar{z})} = \beta e^{a_D(n-1)+[(1-\rho_z)+b_D(n-1)\rho_z](z_t-\bar{z})-(1-\gamma)(1-b_D(n-1))\sigma_{Cz}+(1-b_D(n-1))^2|\sigma_z|^2} \times \frac{1+p((1-\eta)^{1+\xi(1-b_D(n-1))-\gamma}-1)}{1+p((1-\eta)^{1-\gamma}-1)}. \quad \text{(D.12)}$$

Taking logs and collecting terms in $z_t - \bar{z}$ implies the recursion

$$b_D(n) = (1-\rho_z) + \rho_z b_D(n-1) = (1-\rho_z)\sum_{j=0}^{n-1}\rho_z^j = 1-\rho_z^n, \quad \text{(D.13)}$$

while taking logs and collecting constants implies

$$a_D(n) = a_D(n-1) + \log\beta - (1-\gamma)\rho_z^{n-1}\sigma_{Cz} + \rho_z^{2(n-1)}|\sigma_z|^2 + \log(1+p((1-\eta)^{1+\rho_z^{n-1}\xi-\gamma}-1)) - \log(1+p((1-\eta)^{1-\gamma}-1)). \quad \text{(D.14)}$$

(Note that in the limit as $n \to \infty$ we have $a_D(n)/n \to \log\beta$ and $b_D(n)/n \to 0$.) Given this solution, it follows that the price-dividend ratio on the whole market is given by

$$\kappa_D(z_t) = \sum_{n=1}^{\infty}\kappa_D(n, z_t). \quad \text{(D.15)}$$

D.3 Bond prices

Consider the n-period zero-coupon nominal bond, which is a claim to the cash flow Π_{t+n}^{-1} (in real terms). This bond has price $P_{\pi nt} = \Pi_t^{-1}\kappa_\pi(n, q_t)$ and satisfies

$$\kappa_\pi(n, q_t) = \mathbb{E}_t\left[M_{t+1}\frac{\Pi_t}{\Pi_{t+1}}\kappa_\pi(n-1, q_{t+1})\right].$$

Conjecture that

$$\kappa_\pi(n, q_t) = \exp\{a_\pi(n) + b_\pi(n)(q_t - \bar{q})\}$$

for some functions $a_\pi(n)$ and $b_\pi(n)$ with $a_\pi(0) = b_\pi(0) = 0$. Noting that

$$M_{t+1}\frac{\Pi_t}{\Pi_{t+1}} = \beta e^{-\mu-(1-\gamma)^2|\sigma_C|^2/2-q_t-(\sigma_\pi+\gamma\sigma_C)^\top\varepsilon_{t+1}}\frac{(1-\lambda\chi_{t+1})(1-\chi_{t+1})^{-\gamma}}{1+p((1-\eta)^{1-\gamma}-1)}$$

and

$$\kappa_\pi(n-1, q_{t+1}) = \exp\left\{a_\pi(n-1) + b_\pi(n-1)(\rho_q(q_t-\bar{q}) + \sigma_q^\top\varepsilon_{t+1})\right\},$$

this means that

$$e^{a_\pi(n)+b_\pi(n)(q_t-\bar{q})} = \beta e^{a_\pi(n-1)-\mu-\bar{q}+[-1+b_\pi(n-1)\rho_q](q_t-\bar{q})-(1-\gamma)^2|\sigma_C|^2/2+|b_\pi(n-1)\sigma_q-\sigma_\pi-\gamma\sigma_C|^2/2}$$
$$\times\frac{1+p((1-\lambda\eta)(1-\eta)^{-\gamma}-1)}{1+p((1-\eta)^{1-\gamma}-1)}.$$

Taking logs and collecting terms in $q_t - \bar{q}$ implies the recursion

$$b_\pi(n) = -1 + \rho_q b_\pi(n-1) = -\sum_{j=0}^{n-1}\rho_q^j = -\frac{1-\rho_q^n}{1-\rho_q},$$

while taking logs and collecting constants implies

$$a_\pi(n) = a_\pi(n-1) + \log\beta - \mu - \bar{q} - \frac{1}{2}(1-\gamma)^2|\sigma_C|^2 + \frac{1}{2}\left|\frac{1-\rho_q^{n-1}}{1-\rho_q}\sigma_q + \sigma_\pi + \gamma\sigma_C\right|^2$$
$$+\log(1+p((1-\lambda\eta)(1-\eta)^{-\gamma}-1)) - \log(1+p((1-\eta)^{1-\gamma}-1)),$$

or, in terms of the risk-free rate,

$$a_\pi(n) = a_\pi(n-1) - \log R_f - \bar{q} + \frac{1}{2}\left(\left(\frac{1-\rho_q^{n-1}}{1-\rho_q}\right)^2|\sigma_q|^2 + |\sigma_\pi|^2\right) + \frac{1-\rho_q^{n-1}}{1-\rho_q}(\sigma_{q\pi} + \sigma_{Cq}) + \gamma\sigma_{C\pi}$$
$$+\log(1+p((1-\lambda\eta)(1-\eta)^{-\gamma}-1)) - \log(1+p((1-\eta)^{-\gamma}-1)).$$

D.4 Calibration

Table D.1 provides additional detail on our calibration. As Section 2.7 explains, we use I/B/E/S forecasts to proxy for expected dividend growth. Specifically, the de-meaned log of our constructed LTG variable is set to match $(1-\rho_z)(z_t-\bar{z})$. The long-run mean $\bar{z}$ is set to the average of the log of consumption divided by earnings. Expected inflation is calibrated to match median 1-year-ahead inflation projections from the Survey of Professional Forecasters. The persistence and long-run mean of expected inflation are estimated using ordinary least squares on the relationship in Equation (19).

We estimate the variance-covariance matrix of the shocks using the residuals from (16–19), where realized is consumption is taken from real personal consumption expenditures from the Federal Reserve Economic Data (FRED) and where realized inflation is from FRED series CPIAUCSL. The correlation of realized consumption and inflation is allowed to vary between the first and second sample, whereas all other correlations and the volatility remains the same throughout the sample period. We assume no disasters realize in-sample. Note that while μ, for example, appears as a constant in (17), it varies between the first and second sample period.

E International evidence

This appendix applies our model to countries other than the United States. We start by understanding whether a rise in the probability of disaster is an attractive explanation for the decline in government bond yields in these other countries with results presented in Table E.2. Note that we do not have survey expectations data for these countries, so instead of the ex ante inflation-adjusted yield $\bar{y}_b$ from (14), we calibrate to the ex post inflation-adjusted yield, defined in (15).

In all three calibrations, we see that both the discount factor β and the probability of

Table D.1: Parameters for generalized endowment economy

Parameter	Value	Source
Preference parameters		
Risk aversion γ	5	Calibrated
EIS ψ	1	Calibrated
Consumption process		
Disaster probability p	0.0210	Calibrated
Disaster magnitude η	0.30	Calibrated
Consumption shock volatility σ_C	0.016	FRED: A794RX0Q048SBEA
Dividend claim process		
Average consumption-dividend ratio $\bar{z}$	5.072	FRED: PCE and Shiller
Persistence of consumption-dividend ratio ρ_z	0.850	Calibrated
Consumption-dividend ratio shock volatility σ_z	0.261	I/B/E/S
Leverage ξ	2	Calibrated
Inflation process		
Persistence of expected inflation ρ_q	0.942	Survey of Professional Forecasters
Expected inflation shock volatility σ_q	0.003	Survey of Professional Forecasters
Realized inflation shock volatility σ_π	0.014	FRED: CPIAUCSL
Correlations		
$\text{Corr}(\pi, q)$	0.373	Various
$\text{Corr}(C, z)$	0.307	Various
$\text{Corr}(C, q)$	0.114	Various

This table shows parameters for the generalized endowment model which includes rare disasters and inflationary default (see also Table 3). Standard deviations and correlations refer to second moments of shocks. q represents expected inflation outside of disasters. Parameters are in annual terms.

disaster must rise substantially in order to match the data. The discount factor rises by 1.8 pp in the United Kingdom, 0.5 pp in Japan, and 1.9 pp in the other countries. Similarly, the probability of disaster rises from 1.3% to 5.1% in the United Kingdom, 0.6% to 2.4% in Japan, and 0.7% to 3.2% for the other countries. This is consistent with the results in the

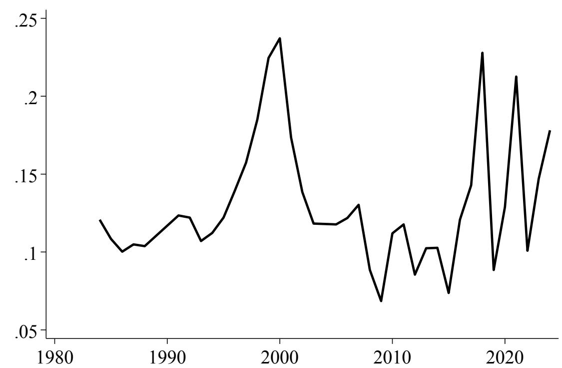

Figure D.2: Analysts expectations for long-term growth

This figure shows the aggregate long-term growth (LTG) measure constructed from I/B/E/S data. Analysts forecast 3–5 year annual growth rates for individual firms, which we aggregate to create an aggregate market forecast for long-term earnings growth.

Alt text: Line graph showing aggregate long-term growth expectations from I/B/E/S analyst forecasts from 1984 to 2024.

United States.

Next, we seek to understand whether the model with disasters and inflationary default can match the evidence from abroad. Here, we find quite similar results to those in the United States. The results are presented in Table E.3. Again, crucially, we see that substantially smaller rises in the discount factor β are needed to match the data. Instead, small declines in the risk of inflationary default are sufficient. Again, this is quite consistent with the results in the United States.

Table E.2: Disaster model calibrated to international data

	Parameter	Values 1984–2000	Values 2001–2020
Panel A: United Kingdom calibration			
Data:			
Price-dividend ratio	κ	27.78	29.46
Ex post inflation-adjusted Treasury yield	$\hat{y}_b$	0.0525	0.0028
Average consumption growth	μ	0.0303	0.0114
Model:			
Discount factor	β	0.954	0.972
Probability of disaster	p	0.0134	0.0510
Panel B: Japan calibration			
Data:			
Price-dividend ratio	κ	138.26	62.80
Ex post inflation-adjusted Treasury yield	$\hat{y}_b$	0.0268	-0.0006
Average consumption growth	μ	0.0252	0.0056
Model:			
Discount factor	β	0.982	0.987
Probability of disaster	p	0.0055	0.0235
Panel C: All other countries calibration			
Data:			
Price-dividend ratio	κ	42.56	51.22
Ex post inflation-adjusted Treasury yield	$\hat{y}_b$	0.0462	0.0008
Average consumption growth	μ	0.0303	0.0099
Model:			
Discount factor	β	0.964	0.983
Probability of disaster	p	0.0073	0.0316

This table shows the parameters necessary to match the data, assuming an endowment economy with rare disasters. We take average consumption growth from the data in each sample. We calibrate the discount factor β and the probability of disaster p to match the average price-dividend ratio and the average inflation-adjusted Treasury bill. We do this for the United Kingdom, Japan, and for countries other than Japan, United Kingdom, and the U.S. that are present in the Jorda-Schularick-Taylor Macrohistory Database, using a population-weighted average of the price-dividend ratio, interest rate, and GDP growth rate. The countries are Australia, Belgium, Canada, Germany, Denmark, Spain, Finland, France, Ireland, Italy, the Netherlands, Norway, Portugal, and Sweden. Parameters and yields are in annual terms.

Table E.3: Inflationary default model calibrated to international data

	Parameter	Values 1984–2000	Values 2001–2020
Panel A: United Kingdom calibration			
Data:			
Price-dividend ratio	κ	27.78	29.46
Ex post inflation-adjusted Treasury yield	$\hat{y}_b$	0.0525	0.0028
Average consumption growth	μ	0.0303	0.0114
Model:			
Discount factor	β	0.958	0.969
Fraction of bond value lost	$\lambda\eta$	0.187	-0.065
Panel B: Japan calibration			
Data:			
Price-dividend ratio	κ	138.26	62.80
Ex post inflation-adjusted Treasury yield	$\hat{y}_b$	0.0268	-0.0006
Average consumption growth	μ	0.0252	0.0056
Model:			
Discount factor	β	0.988	0.990
Fraction of bond value lost	$\lambda\eta$	0.252	0.111
Panel C: All other countries calibration			
Data:			
Price-dividend ratio	κ	42.56	51.22
Ex post inflation-adjusted Treasury yield	$\hat{y}_b$	0.0462	0.0008
Average consumption growth	μ	0.0303	0.0099
Model:			
Discount factor	β	0.970	0.984
Fraction of bond value lost	$\lambda\eta$	0.237	0.055

This table shows the parameters necessary to match the data, assuming an endowment economy with rare disasters and inflationary default. We take average consumption growth from the data in each sample. We calibrate the discount factor β and the decline in bond value $\lambda\eta$ to match the average price-dividend ratio and the average inflation-adjusted Treasury bill, assuming no disasters in-sample. We assume the disaster probability equals 2.10%. We do this for the United Kingdom, Japan, and for countries other than Japan, United Kingdom, and the U.S. that are present in the Jorda-Schularick-Taylor Macrohistory Database, using a population-weighted average of the price-dividend ratio, interest rate, and GDP growth rate. The countries are Australia, Belgium, Canada, Germany, Denmark, Spain, Finland, France, Ireland, Italy, the Netherlands, Norway, Portugal, and Sweden. Parameters and yields are in annual terms.

F Government debt and inflationary default risk

Another question that arises from our model is how it can be possible that sovereign default risk could go down when government debt-to-GDP has risen so much over the same period. To answer this question, we use the fiscal theory of the price level (Cochrane 2023) to endogenize the inflation process in terms of fiscal policy. This allows us to show how the risk of sovereign default (the parameter λ) and the present value of government debt depend on beliefs about future surpluses.

Suppose the government runs surpluses $S_t = T_t - G_t$ and issues one-period debt with total nominal value $Q_t^{\$} B_t$. The government budget constraint is

$$B_{t-1} = \Pi_t S_t + Q_t^{\$} B_t. \tag{F.1}$$

The left-hand side is the face value of bonds owed to investors (each bond issued at $t-1$ promises \$1 at time t). The government fulfills this obligation by a combination of paying off the debt (the term $\Pi_t S_t$, representing the nominal value of the surplus) and issuing new debt B_t at price $Q_t^{\$}$. Iterating this constraint forward and taking expectations implies the present-value relation

$$\frac{B_{t-1}}{\Pi_t} = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t+j} S_{t+j} \right]. \tag{F.2}$$

The price level Π_t, and hence the inflation process, is determined by this equation.

For convenience, let us write the surplus as the product of current consumption and the surplus-consumption ratio,

$$S_t = s_t C_t. \tag{F.3}$$

To highlight the importance of disaster default in our model, we assume that the surplus-consumption ratio follows a three-state process that depends on the occurrence of a disaster:

$$s_t = \begin{cases} \bar{s} & \text{if} \quad \chi_{t-1} = \chi_t = 0, \\ \bar{s} - s^- & \text{if} \quad \chi_{t-1} = 0 \quad \text{and} \quad \chi_t = \eta, \\ \bar{s} + s^+ & \text{if} \quad \chi_{t-1} = \eta. \end{cases} \tag{F.4}$$

In nondisaster times, the government raises a constant fraction $\bar{s}$ of the endowment as surpluses. When a disaster occurs, the government runs a deficit s^-, but commits to repaying that deficit at a rate s^+ thereafter. The higher the disaster-contingent repayment rate s^+, the less the government will need to default by inflation in a disaster.

Let us now define the debt-consumption (i.e., debt-to-GDP) ratio

$$b_t \equiv \frac{B_{t-1}}{\Pi_t C_t}.$$

We can rewrite the valuation equation (F.2) in terms of this ratio recursively as

$$b_t = s_t + \mathbb{E}_t \left[M_{t+1} \frac{C_{t+1}}{C_t} b_{t+1} \right]. \tag{F.5}$$

Assuming the agent has risk aversion γ and a unit EIS, we can next substitute in the stochastic discount factor (SDF), and this relation becomes

$$b_t = s_t + \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right]^{-1} \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} b_{t+1} \right]. \tag{F.6}$$

Finally, substituting in the endowment process,

$$b_t = s_t + \beta \mathbb{E}_t \left[\frac{(1-\chi_{t+1})^{1-\gamma}}{(1-p) + p(1-\eta)^{1-\gamma}} b_{t+1} \right]. \tag{F.7}$$

Now, because $s_t = s(\chi_{t-1}, \chi_t)$, we have a solution of the form $b_t = b(\chi_{t-1}, \chi_t)$, which solves the system of equations

$$b(\chi_{t-1}, \chi_t) = s(\chi_{t-1}, \chi_t) + \beta \mathbb{E}_t[\tilde{p} b(\chi_t, \eta) + (1-\tilde{p}) b(\chi_t, 0)], \tag{F.8}$$

where

$$\tilde{p} \equiv \frac{p(1-\eta)^{1-\gamma}}{(1-p) + p(1-\eta)^{1-\gamma}} \tag{F.9}$$

is a risk-adjusted disaster probability (equal to the physical probability p if $\gamma = 1$). This is a linear system of four equations in four unknowns $b(\chi_{t-1}, \chi_t)$, the solution to which is

$$\begin{bmatrix} b(\eta,0) \\ b(0,\eta) \\ b(\eta,\eta) \\ b(0,0) \end{bmatrix} = \begin{bmatrix} 1 & -\beta\tilde{p} & 0 & -\beta(1-\tilde{p}) \\ -\beta(1-\tilde{p}) & 1 & -\beta\tilde{p} & 0 \\ -\beta(1-\tilde{p}) & 0 & 1-\beta\tilde{p} & 0 \\ 0 & -\beta\tilde{p} & 0 & 1-\beta(1-\tilde{p}) \end{bmatrix}^{-1} \begin{bmatrix} \bar{s} + s^+ \\ \bar{s} - s^- \\ \bar{s} + s^+ \\ \bar{s} \end{bmatrix}. \tag{F.10}$$

Computing the matrix inverse and multiplying by the surplus vector, we then get the explicit solution

$$\begin{bmatrix} b(\eta,0) \\ b(0,\eta) \\ b(\eta,\eta) \\ b(0,0) \end{bmatrix} = \frac{1}{1-\beta}\bar{s} + \begin{bmatrix} \beta^2\tilde{p} - \beta + 1 \\ \beta(\beta\tilde{p} - \beta + 1) \\ \beta^2\tilde{p} - \beta^2 + 1 \\ \beta^2\tilde{p} \end{bmatrix} \frac{1}{1-\beta} s^+ - \begin{bmatrix} \beta\tilde{p}(1-\beta\tilde{p}) \\ \beta^2\tilde{p}(1-\tilde{p}) + 1 - \beta \\ \beta^2\tilde{p}(1-\tilde{p}) \\ \beta\tilde{p}(1-\beta\tilde{p}) \end{bmatrix} \frac{1}{1-\beta} s^-. \tag{F.11}$$

Now let us use this solution to determine the equilibrium inflation process. Suppose there have been no recent disasters, so $b_t = b(0,0)$. In this case, we have

$$b(0,0) = \frac{1}{1-\beta}\bar{s} + \frac{\beta^2\tilde{p}}{1-\beta} s^+ - \frac{\beta\tilde{p}(1-\beta\tilde{p})}{1-\beta} s^-. \tag{F.12}$$

Next period's debt-to-consumption will either remain at $b(0,0)$ or become

$$b(0,\eta) = \frac{1}{1-\beta}\bar{s} + \frac{\beta^2\tilde{p} + \beta(1-\beta)}{1-\beta}s^+ - \frac{(1-\beta(1-\tilde{p}))(1-\beta\tilde{p})}{1-\beta}s^- \tag{F.13}$$

if there is a disaster. Thus, we can write

$$b(0,\eta) = b(0,0) + \beta s^+ - (1-\beta\tilde{p})s^-, \tag{F.14}$$

or, in terms of the disaster shock,

$$b_{t+1} = b(0,0) - \left[(1-\beta\tilde{p})s^- - \beta s^+\right]\frac{\chi_{t+1}}{\eta}. \tag{F.15}$$

To solve for the inflation rate, note that the flow budget constraint can be rewritten

$$Q_t^\$ \frac{\Pi_{t+1}}{\Pi_t} = \left(\frac{C_{t+1}}{C_t}\right)^{-1} \frac{b_t - s_t}{b_{t+1}}. \tag{F.16}$$

Now conjecture that there exists a constant λ and a deterministic process $\mu_{\pi t}$ such that

$$\frac{\Pi_{t+1}}{\Pi_t} = e^{\mu_{\pi t}}(1-\lambda\chi_{t+1})^{-1}. \tag{F.17}$$

Substituting this conjecture in (note that $Q_t^\$ = e^{-\mu_{\pi t}-\bar{y}_b}$) and collecting disaster terms implies

$$1-\lambda\chi_{t+1} = (1-\chi_{t+1})\left(1 - \frac{(1-\beta\tilde{p})s^- - \beta s^+}{\eta b(0,0)}\chi_{t+1}\right). \tag{F.18}$$

Expanding the right-hand side and using the fact that $\chi_{t+1}^2 = \eta\chi_{t+1}$,[4] this implies

$$1-\lambda\chi_{t+1} = 1 - \left(1 - \left(\frac{1-\eta}{\eta}\right)\left(\frac{\beta s^+ - (1-\beta\tilde{p})s^-}{b(0,0)}\right)\right)\chi_{t+1}, \tag{F.19}$$

[4]Recall that χ_{t+1} equals either 0 or η. If $\chi_{t+1} = 0$, then $\chi_{t+1}^2 = 0$. If $\chi_{t+1} = \eta$, then $\chi_{t+1}^2 = \eta^2 = \eta\chi_{t+1}$. Thus, $\chi_{t+1}^2 = \eta\chi_{t+1}$ for both possible outcomes.

or, equivalently, that

$$\lambda = 1 - \left(\frac{1-\eta}{\eta}\right)\left(\frac{\beta s^+ - (1-\beta\tilde{p})s^-}{b(0,0)}\right). \tag{F.20}$$

Finally, noting that (F.12) implies

$$\beta s^+ - (1-\beta\tilde{p})s^- = \frac{(1-\beta)b(0,0) - \bar{s}}{\beta\tilde{p}}, \tag{F.21}$$

we can write λ in terms of the debt-to-GDP ratio $b(0,0)$:

$$\lambda = \left(1 - \left(\frac{1-\eta}{\eta}\right)\frac{1-\beta}{\beta\tilde{p}}\right) + \left(\frac{1-\eta}{\eta}\right)\frac{1}{\beta\tilde{p}}\frac{\bar{s}}{b(0,0)}. \tag{F.22}$$

For any set of parameters, the default size λ is inversely related to the debt-to-GDP ratio. Intuitively, this is because the government's debt is more valuable when investors believe that the risk of default is lower.

All else equal, the debt-to-GDP ratio b is decreasing in the size of the disaster deficit s^-, but increasing in the repayment rate s^+. By (F.22), the magnitude of the inflation in a disaster state, λ, is therefore increasing in s^- and decreasing in s^+. Figure F.3 plots a comparative static for the equilibrium default size $\eta\lambda$ and the debt-to-GDP ratio b_t as a function of the expected repayment rate s^+. Panel A shows that, as expected repayment s^+ increases, $\eta\lambda$ falls. This suggests one possible explanation for the decline in λ we estimate in the data: growing confidence that the government will pay back investors should a disaster occur. Importantly, what matters for this fiscal-theoretic view of the price level is beliefs about future surpluses. In particular, what matters here is not so much the normal-times surplus rate $\bar{s}$, but beliefs about the surplus rate s^+ following disasters. Data on beliefs about disaster-contingent repayment are not available to directly test this hypothesis, but Jiang et al. (2024) do present evidence that expectations of future surpluses in the twenty-first century were high, and indeed much higher than the surpluses that actually materialized, consistent with this story.

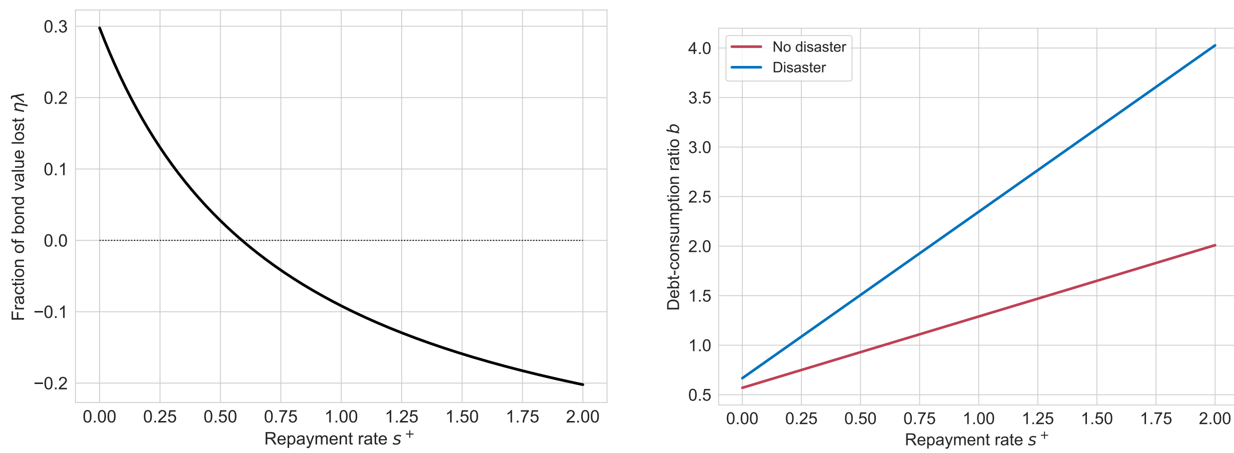

Figure F.3: Government debt and disaster inflation in the fiscal theory

The figure shows comparative statics with respect to the post-disaster repayment rate s^+ from the surplus process (F.4). Both panels hold fixed all other model parameters, including the other surplus parameters $\bar{s}$ and s^-. Panel A shows the fraction of nominal bond value lost in a disaster, $\lambda\eta$, as a function of this parameter. Panel B shows the debt-consumption ratio in the state without any disasters ($b(0,0)$, the red line) and during a disaster ($b(0,\eta)$, the blue line).

Alt text: Two-panel figure showing theoretical relationships in the fiscal theory. Panel A shows bond value lost as a function of repayment rate. Panel B shows debt-consumption ratios for disaster and no-disaster scenarios.

To address the question of how high debt levels could co-occur with low default risk $\eta\lambda$, Panel B plots debt-to-GDP both in the nondisaster ($b(0,0)$) and disaster ($b(0,\eta)$) states as a function s^+. In both states, the value of debt-to-GDP is increasing in the expected repayment rate for two reasons. First is a cashflow effect, whereby higher repayment means higher surpluses on average. Second is a risk premium effect, whereby higher repayment means a larger increase in the value of government debt in the disaster state (the blue line), rendering government debt a hedge against disaster risk and increasing its value ex ante. Notably, the increased fiscal capacity from high s^+ could allow the government some flexibility to reduce its normal-times surplus rate $\bar{s}$, which could also help explain how debt continued to rise amid unexpectedly low surpluses.

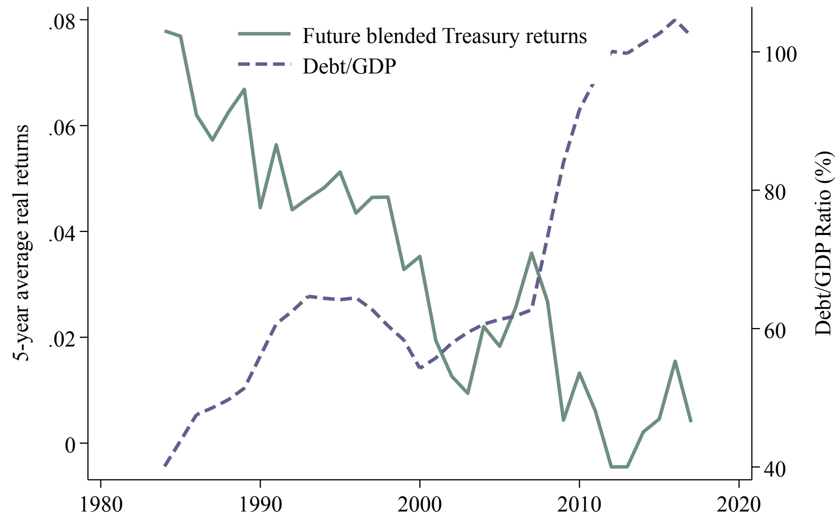

Figure F.4: Government debt level and returns in the data

The figure shows the time series of total real returns on government debt and the government debt-to-GDP ratio from 1984 to 2016. The solid green line (left axis) is the 5-year-ahead realized real return on the portfolio of all U.S. Treasuries. This blended return series is computed as in Jiang et al. (2024). The dashed purple line (right axis) is the ratio of total debt to GDP in the U.S.

Alt text: Dual-axis line graph showing the relationship between the debt-to-GDP ratio and future Treasury returns from 1980 to 2020.

Figure F.4 plots the trends in government debt-to-GDP (the right axis) and real returns on government debt (the left axis) over our sample period. For returns, we compute the total real return on all Treasuries following Jiang et al. (2024) and then average those returns over the next 5 years. Clearly, as debt-to-GDP has increased, real returns on debt have fallen, suggesting a decline in the overall discount rate (including the default risk premium). Importantly, as Jiang et al. (2024) show, the decline in the discount rate alone is not enough to explain the rise in debt, and so it remains a puzzle that debt has risen amid persistent deficits. As we discuss in Section 2.5, our main point in doing this analysis is to say that rising debt and declining risk of default are not in contradiction. It is not to say that lower inflation risk alone explains the rise in debt. The stylized model above assumes a simple, stationary model of surpluses and imposes rational expectations; in reality, investors' beliefs about the surplus process may be non-stationary and may not accord with the data-generating process, and these deviations are likely to be important for explaining the rise in debt along with the decline in default risk.

The only substantive assumptions on the surplus process that we require is that, in the event of a consumption disaster, news is revealed about the path of future surpluses. If this news is negative (namely, unexpected deficits that will not be repaid), then default (inflation) will occur if and when a disaster occurs. Aside from this assumption, the surplus process can have arbitrary, even non-stationary, dynamics outside of disasters, and our main point will still stand.

G Production model

G.1 Solution to the no-inventory case

Consider the model in Section 3.1. The agent maximizes (28), subject to (27). Conjecture that

$$V(W_t) = \nu W_t, \tag{G.1}$$

for some constant $\nu > 0$. Substituting this conjecture into (28), with $R_{W,t+1} \equiv R_{f,t+1} + \alpha_t(R_{K,t+1} - R_{f,t+1})$ implies

$$(1-\beta)\log\nu + \log W_t = \max_{C_t,\alpha_t}\left\{(1-\beta)\log C_t + \beta\log\left(W_t - C_t\right) + \frac{\beta}{1-\gamma}\log\left(\mathbb{E}_t\left[R_{W,t+1}^{1-\gamma}\right]\right)\right\}. \tag{G.2}$$

At the optimum, the derivative of the right-hand side with respect to C_t equals zero. Thus:

$$\frac{1-\beta}{C_t} - \frac{\beta}{W_t - C_t} = 0$$

yielding the result $C_t/W_t = 1 - \beta$. Moreover, setting the derivative of the right hand side with respect to α equal to zero yields

$$\mathbb{E}_t \left[R_{W,t+1}^{1-\gamma}\right]^{-1} \mathbb{E}_t \left[R_{W,t+1}^{-\gamma}(R_{K,t+1} - R_{f,t+1})\right] = 0. \tag{G.3}$$

From these conditions, we can derive growth and asset prices on a balanced-growth path (i.e., when consumption, investment, and output grow at the same rate). First, note that wealth grows at rate:

$$\frac{W_{t+1}}{W_t} = \frac{W_t - C_t}{W_t}\frac{W_{t+1}}{W_t - C_t} = \beta R_{K,t+1}. \tag{G.4}$$

(We have used the constant consumption-wealth ratio and the equilibrium condition $\alpha = 1$.) This must also be the growth rate of consumption. Substituting in for $R_{K,t+1}$ implies

$$\frac{C_{t+1}}{C_t} = \beta(1 - \delta + A)(1 - \chi_{t+1}). \tag{G.5}$$

This is then also the growth rate of planned capital, lagged one period. In equilibrium, all investment is in planned capital, so $\tilde{K}_{t+1}/\tilde{K}_t = W_t/W_{t-1}$. From (G.4), then, it follows that[5]

$$\frac{K_{t+1}}{K_t} = \frac{\tilde{K}_{t+1}}{\tilde{K}_t}\frac{1-\chi_{t+1}}{1-\chi_t} = \beta R_{K,t}\frac{1-\chi_{t+1}}{1-\chi_t} = \beta(1-\delta+A)(1-\chi_{t+1}).$$

The result for output then follows from $Y_t = AK_t$ and the result for investment follows from (29). As a consequence, the price-dividend ratio κ^Y on the claim to output equals the price-dividend ratio κ on the consumption claim: $\kappa^Y = \kappa = \beta/(1-\beta)$.

We now turn to the implications of this model for the interest rate and for stock returns. The equilibrium condition $\alpha = 1$ implies $R_{W,t+1} = R_{K,t+1}$. Substituting $R_{K,t+1}$ into (G.3)

[5]Here we have used the fact that planned capital $\tilde{K}_{t+1}$ is a constant fraction of wealth W_t (in this model, this fraction is one), so that $\tilde{K}_{t+1}/\tilde{K}_t = W_t/W_{t-1} = \beta R_{K,t}$.

implies a value for the log risk-free rate:

$$\begin{aligned}\log R_f &= \log \mathbb{E}_t \left[R_{K,t+1}^{1-\gamma}\right] - \log \mathbb{E}_t \left[R_{K,t+1}^{-\gamma}\right] \\ &= \log(1-\delta+A) + \log(1+p((1-\eta)^{1-\gamma}-1)) - \log(1+p((1-\eta)^{-\gamma}-1)) \\ &\approx A - \delta + p((1-\eta)^{1-\gamma} - (1-\eta)^{-\gamma}).\end{aligned} \tag{G.6}$$

Equations (G.3) and (G.6) imply the following expression for the SDF:

$$M_{t+1} = \mathbb{E}_t \left[R_{W,t+1}^{1-\gamma}\right]^{-1} R_{W,t+1}^{-\gamma}. \tag{G.7}$$

Furthermore, the risk premium equals

$$\begin{aligned}\log \mathbb{E}_t[R_{K,t+1}] - \log R_f = \log(1-p\eta) \\ + \log(1+p((1-\eta)^{-\gamma}-1)) - \log(1+p((1-\eta)^{1-\gamma}-1)),\end{aligned} \tag{G.8}$$

exactly as in the endowment economy.

Note that, as discussed in the main text, asset prices are identical to those in the endowment economy if $\beta^{-1}e^{\mu} = (1-\delta+A)$. The key difference between the models, however, is that there are two margins of adjustment in the production economy: quantities and prices. This is why, for example, the patience parameter β does not show up in (G.6). Instead, β influences quantities through the investment-capital ratio, which in turn affects prices. In the standard endowment economy, quantities cannot adjust, as the representative investor consumes whatever is produced in a given period.

G.2 Solution to the general case

The agent can invest in an inventory asset with net return $r_I = 0$, a risk-free bond with net return $r_{f,t+1}$, and a risky capital asset with net return $r_{K,t+1}$. Let $r_{j,t+1}$, $j \in \mathcal{J} =$

$\{I, f, K\}$, represent net returns, and let $\alpha_{j,t}$ denote the percent allocation of savings to asset j. Note that, in our setting with a binary shock χ_{t+1}, markets are complete, so the agent will be able to construct any state-contingent portfolio return $r_{i,t+1}$. Inventory and capital are the only securities in positive net supply; furthermore, we restrict inventory to be in non-negative supply ($I_t \geq 0$). It follows from this setup that the return on wealth $R_{W,t+1} = \sum_{j \in \mathcal{J}} \alpha_{j,t}(1 + r_{j,t+1})$, where $\sum_{j \in \mathcal{J}} \alpha_{j,t} = 1$.

Suppose that the agent has Epstein-Zin utility with unit EIS. The agent's optimization problem is therefore

$$\max_{C_t, \{\alpha_{j,t}\}_{j \in \mathcal{J}}} \left(C_t^{1-\beta} \left(\mathbb{E}_t \left[V(W_{t+1})^{1-\gamma} \right] \right)^{\frac{\beta}{1-\gamma}} \right), \tag{G.9}$$

subject to the dynamic budget constraint

$$W_{t+1} = (W_t - C_t) R_{W,t+1} = (W_t - C_t) \sum_{j \in \mathcal{J}} \alpha_{j,t}(1 + r_{j,t+1}), \tag{G.10}$$

the portfolio weight restriction

$$\sum_{j \in \mathcal{J}} \alpha_{j,t} = 1, \tag{G.11}$$

and the inventory non-negativity constraint

$$\alpha_{I,t} \geq 0. \tag{G.12}$$

Let ζ_t and ξ_t denote the Lagrange multipliers on the constraints (G.11) and (G.12), respectively.

Substituting (G.1) and the budget constraint (G.10) into (G.9), then taking logs, we again obtain (G.2) and the identical first-order condition for consumption as above. The first-order condition with respect to asset allocation $\alpha_{j,t}$, $j \neq I$, is

$$\beta \mathbb{E}_t \left[R_{W,t+1}^{1-\gamma} \right]^{-1} \mathbb{E}_t \left[R_{W,t+1}^{-\gamma} (1 + r_{j,t+1}) \right] = \zeta_t, \tag{G.13}$$

and the first-order condition with respect to the inventory allocation $\alpha_{I,t}$ is

$$\beta \mathbb{E}_t \left[R_{W,t+1}^{1-\gamma}\right]^{-1} \mathbb{E}_t \left[R_{W,t+1}^{-\gamma}\right] + \xi_t = \zeta_t. \tag{G.14}$$

Multiply both sides of (G.13) by $\alpha_{j,t}$, take the sum over $j \in \mathcal{J} \setminus \{I\}$, and substitute in (G.14) to see that

$$\zeta_t = \beta + \xi_t \alpha_{I,t} = \beta, \tag{G.15}$$

by complementary slackness. This implies the Euler equation for gross returns

$$\mathbb{E}_t \left[R_{W,t+1}^{1-\gamma}\right]^{-1} \mathbb{E}_t \left[R_{W,t+1}^{-\gamma} R_{j,t+1}\right] = 1 \tag{G.16}$$

and the Euler equation for inventory

$$\mathbb{E}_t \left[R_{W,t+1}^{1-\gamma}\right]^{-1} \mathbb{E}_t \left[R_{W,t+1}^{-\gamma}\right] + \frac{\xi_t}{\beta} = 1. \tag{G.17}$$

Note the market clearing condition $\alpha_{I,t} = 1 - \alpha_{K,t}$, where $\alpha_{K,t}$ is simply denoted α_t in our setup in the main text. We thus have that $\xi_t > 0$ if and only if $\alpha_t < 1$.

We now show formally that inventory imposes a zero lower bound. Throughout, we assume that the bond is in zero net supply.

Lemma 1. *If $\alpha_t < 1$, then the gross real risk-free rate $R_{f,t+1} = 1$. If $\alpha_t = 1$, then $R_{f,t+1} \geq 1$ and is equal to the real risk-free rate in a no-inventory economy $R^*_{f,t+1}$.*

Proof. If $\alpha_{I,t} > 0$, then $\xi_t = 0$ and (G.16) and (G.14) combine to give us $R_{f,t+1} = 1$. If $\alpha_{I,t} = 0$, then $\xi_t \geq 0$ and

$$R_{f,t+1} = \frac{\beta}{\beta - \xi_t}, \tag{G.18}$$

which is greater than or equal to 1. Moreover, if $\alpha_{I,t} = 0$, then market clearing implies

$R_{W,t+1} = R_{K,t+1}$ and the Euler equation (G.16) yields

$$R_{f,t+1} = \mathbb{E}_t \left[R_{K,t+1}^{1-\gamma}\right] \mathbb{E}_t \left[R_{K,t+1}^{-\gamma}\right]^{-1}, \tag{G.19}$$

which is the same as the risk-free rate $R^*_{f,t+1}$ in the no-inventory economy. ■

We next show that the unconstrained risk-free rate determines α.

Theorem 1. *If the unconstrained gross risk-free rate $R^*_{f,t+1} < 1$, then $\alpha_t < 1$ and the constrained risk-free rate $R_{f,t+1} = 1$. If $R^*_{f,t+1} \geq 1$, then $\alpha_t = 1$ and the equilibrium is as in a standard no-inventory production economy with $R_{f,t+1} = R^*_{f,t+1}$.*

Proof. We will prove the theorem by contradiction using Lemma 1.

Suppose $R^*_{f,t+1} < 1$ and $\alpha_{I,t} = 0$. Then $R_{f,t+1} = R^*_{f,t+1} < 1$, which contradicts Lemma 1. It must therefore be the case that $R^*_{f,t+1} < 1$ implies $\alpha_{I,t} > 0$, which implies $R_{f,t+1} = 1$.

Now suppose $R^*_{f,t+1} > 1$ and $\alpha_{I,t} > 0$. Then $R_{f,t+1} = 1 < R^*_{f,t+1}$, which contradicts Lemma 1. Moreover, in the knife-edge case $R^*_{f,t+1} = 1$, the equilibrium conditions (G.16) and (G.14) imply $\xi_t = 0$, which implies that $\alpha_{I,t} = 0$ and $R_{f,t+1} = R^*_{f,t+1} = 1$. Thus, it must be that $R^*_{f,t+1} \geq 1$ implies $\alpha_{I,t} = 0$, which implies $R_{f,t+1} = R^*_{f,t+1} \geq 1$. ■

We conjecture that the price-dividend ratio depends only on the current state χ_t (i.e., whether the disaster occurred or not). The intuition for this is that output growth Y_{t+1}/Y_t is a function of χ_t only. Thus,

$$1 = \mathbb{E}_t \left[R_{W,t+1}^{1-\gamma}\right]^{-1} \mathbb{E}_t \left[R_{W,t+1}^{-\gamma} \left(\frac{\kappa^Y(\chi_{t+1}) + 1}{\kappa^Y(\chi_t)}\right) \left(\frac{Y_{t+1}}{Y_t}\right)\right]. \tag{G.20}$$

This implies that we have two equations, one for the nondisaster state,

$$\kappa^Y(0) = \hat{\beta}\Big[(1-p)(1+\alpha r_{K,0})^{1-\gamma}(\kappa^Y(0)+1) + p(1+\alpha r_{K,\eta})^{-\gamma}(\kappa^Y(\eta)+1)(1-\eta)(1+\alpha r_{K,0})\Big], \quad \text{(G.21)}$$

and one for the disaster state,

$$\kappa^Y(\eta) = \hat{\beta}\Big[(1-p)(1+\alpha r_{K,0})^{-\gamma}(\kappa^Y(0)+1)(1-\eta)^{-1}(1+\alpha r_{K,\eta}) + p(1+\alpha r_{K,\eta})^{1-\gamma}(\kappa^Y(\eta)+1)\Big]. \quad \text{(G.22)}$$

In these equations, $\hat{\beta} \equiv \beta\Big[(1-p)(1+\alpha r_{K,0})^{1-\gamma} + p(1+\alpha r_{K,\eta})^{1-\gamma}\Big]^{-1}$, $r_{K,0} \equiv (1-\delta+A)-1$, and $r_{K,\eta} \equiv (1-\delta+A)(1-\eta)-1$. The solution to this system is as stated in the main text (after defining the weights ν).

Although the price-dividend ratio is state-dependent when the agent chooses to hold inventory, the risk premium is not. The risk premium at time t when the agent holds inventory is given by $\log \mathbb{E}_t[R^Y_{t+1}] - \log R_f$, for the expected return on the output claim

$$\mathbb{E}_t[R_{Y,t+1}] = \mathbb{E}_t\left[\left(\frac{\kappa^Y(\chi_{t+1})+1}{\kappa^Y(\chi_t)}\right)\left(\frac{Y_{t+1}}{Y_t}\right)\right]. \quad \text{(G.23)}$$

If the expected return on the output claim is the same across states, then so is the risk premium. In the no-disaster state, the expected return on the output claim is

$$\mathbb{E}_t[R_{Y,t+1}|\chi_t = 0] = \left(\frac{(1-p)\kappa^Y(0) + p\kappa^Y(\eta)+1}{\kappa^Y(0)}\right)\times \Big(\beta(1-p\eta)\left(\alpha(1-\delta+A)+1-\alpha\right)\Big) \quad \text{(G.24)}$$

and in the disaster state by

$$\mathbb{E}_t[R_{Y,t+1}|\chi_t = \eta] = \left(\frac{(1-p)\kappa^Y(0) + p\kappa^Y(\eta) + 1}{\kappa^Y(\eta)}\right)\times \left(\beta(1-p\eta)\left(\alpha(1-\delta+A) + \left(\frac{1-\alpha}{1-\eta}\right)\right)\right). \quad \text{(G.25)}$$

Examining the two expressions, we see that the expected return in both states are the same if and only if

$$\kappa^Y(\eta)(1-\eta)\bigg(\alpha(1-\delta+A) + 1 - \alpha\bigg) = \kappa^Y(0)\bigg(\alpha(1-\delta+A)(1-\eta) + 1 - \alpha\bigg).$$

The terms inside the parentheses can be written so that

$$\kappa^Y(\eta)(1-\eta)(1+\alpha r_{K,0}) = \kappa^Y(0)(1+\alpha r_{K,\eta}),$$

which is true if we substitute in the expressions for $\kappa^Y(\chi_t)$. This implies that, while the price-dividend ratio is time-varying, the risk premium is not.

Finally, let us solve for the ex ante inflation-adjusted Treasury yield. The price of the one-period nominal bond in this economy is

$$Q_t^{\$} = \mathbb{E}_t\left[R_{W,t+1}^{1-\gamma}\right]^{-1}\mathbb{E}_t\bigg[R_{W,t+1}^{-\gamma}(1-L_{t+1})\bigg]e^{-q_t+\sigma_\pi^2/2},$$

which evaluates to

$$Q_t^{\$} = \frac{p(1+\alpha r_{K,\eta})^{-\gamma}(1-\lambda\eta) + (1-p)(1+\alpha r_{K,0})^{-\gamma}}{p(1+\alpha r_{K,\eta})^{1-\gamma} + (1-p)(1+\alpha r_{K,0})^{1-\gamma}} e^{-q_t+\sigma_\pi^2/2}.$$

Thus, the nominal yield is

$$y^{\$}_{b,t} = -\log Q^{\$}_t = \log(p(1+\alpha r_{K,\eta})^{1-\gamma} + (1-p)(1+\alpha r_{K,0})^{1-\gamma}) \\ - \log(p(1+\alpha r_{K,\eta})^{-\gamma}(1-\lambda\eta) + (1-p)(1+\alpha r_{K,0})^{-\gamma}) + q_t - \frac{1}{2}\sigma^2_\pi,$$

and, by (13), the ex ante inflation-adjusted Treasury yield is

$$\bar{y}_b = \log(p(1+\alpha r_{K,\eta})^{1-\gamma} + (1-p)(1+\alpha r_{K,0})^{1-\gamma}) \\ - \log(p(1+\alpha r_{K,\eta})^{-\gamma}(1-\lambda\eta) + (1-p)(1+\alpha r_{K,0})^{-\gamma}) - \sigma^2_\pi - \log(1+p\lambda\eta(1-\lambda\eta)^{-1}). \tag{G.26}$$

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